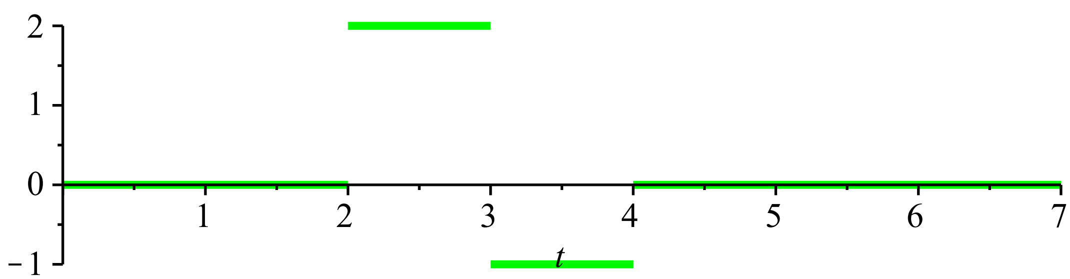
Math 2250-4
Week 12 concepts and homework, due April 6.

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

10.4: using convolution integrals to find inverse Laplace of F(s)G(s); using the differentiation theorem to find the Laplace transform of $t \cdot f(t)$; using these (and other) techniques to solve linear differential equations.
10.4: 2, 3, 9, 15, **26**, 29, **30**, 37, **36**
w12.1a) Compute the convolution $f*g(t)$ for $f(t) = t$ and $g(t) = t^2$ and use the Laplace transform table to show that $\mathcal{L}\{f*g(t)\}(s) = F(s)G(s)$ in this case.
b) Repeat part a for the example $f(t) = t,\ g(t) = \cos(k\,t)$.

10.5: using the unit step function to turn forcing on and off; Laplace transform of non-sinusoidal periodic functions
10.5: 3, 7, 11, 13, 25, 31, **26**, **34**

w12.2) In the week 11 homework problem you solved the following Laplace transform problem using the definition of Laplace transform. This week, write $f(t)$ as a linear combination of unit step functions in order to find (the same) Laplace transform. Here's the graph of f, which is zero for $t \geq 4$:



EP 7.6 impulse functions as limits of differences of scaled unit step functions.
w12.3) Solve this initial value problem and then sketch the solution. Explain what happens. The forcing term is a sum of instananeous unit impulses, as discussed in EP7.6 and Monday's class notes.

$$x''(t) + 4\,x(t) = \delta(t-\pi) + \delta(t-2\,\pi) - 2\,\delta(t-4\,\pi).$$
$$x(0) = 0$$
$$x'(0) = 0$$

6.1: finding eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional).
6.1: 7, 17, 19, 25

6.2: diagonalizability: recognizing matrices for which there is an eigenbasis, and those for which there is not one. Writing the eigenbasis equation as AP = PD where P is the matrix of eigenvector columns, and D is the diagonal matrix of eigenvalues.
6.2: 3, 9, 21, 23.

w12.4) Find eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional), for the following matrices. You can check your work with technology, but you don't have to hand in the technology check.

a) $A := \begin{bmatrix} -8 & -6 \\ 9 & 7 \end{bmatrix}$

b) $B := \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$

c) $C := \begin{bmatrix} 2 & 9 & 3 \\ -2 & -5 & 0 \\ 2 & 6 & 1 \end{bmatrix}$

d) $E := \begin{bmatrix} -1 & 6 & 6 \\ 0 & -3 & -2 \\ 0 & 4 & 3 \end{bmatrix}$

e) $F := \begin{bmatrix} 7 & 3 & -9 \\ -4 & -3 & 4 \\ 4 & 2 & -5 \end{bmatrix}$.

12.5)
a) Which of the matrices in 12.4 are diagonalizable and which are not?
b) For the diagonalizable matrices A, C above, verify the diagonalizing identity $P^{-1} A\, P = \mathrm{D}$ by checking the equivalent equation $A\, P = P\, \mathrm{D}$.
c) Explain what happens to the diagonal matrix D of eigenvalues for a diagonalizable matrix, if you change the order of the eigenvector columns in P, in the identity $A\, P = P\, \mathrm{D}$.

12.6) Compute A^{10} by hand, for the matrix A in 12.4. Hint: use the identity $A = P\, \mathrm{D}\, P^{-1}$. (You can check your answer with technology, but don't have to hand that part in.)