

Math 2250-4

Week 11 concepts and homework, due March 28 **Wednesday**.
Thursday exam will cover this material.

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in. There is no Friday quiz for this week, because of the exam on Thursday.

10.1: Laplace transforms and inverse transforms.

Use the definition of Laplace transform and integration techniques to compute Laplace transforms of $f(t)$:

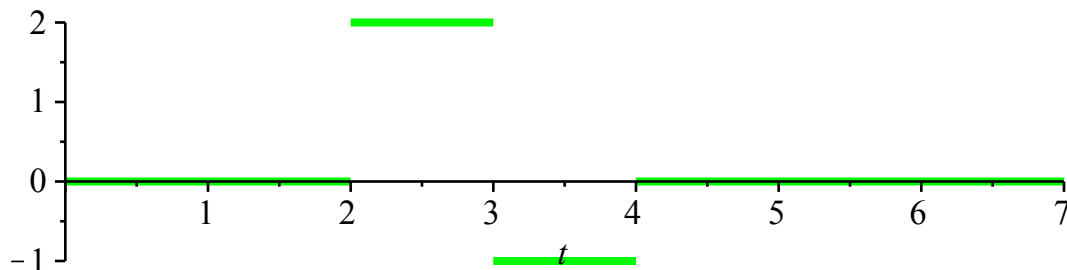
1, 3, 7, 9

w11.1) Use the definition of Laplace transform

$$\mathcal{L}\{f(t)\}(s) = \int_0^{\infty} f(t)e^{-st} dt$$

to compute the Laplace transform of $f(t) = t e^{2t}$.

w11.2) Consider the function $f(t)$ which is zero for $t > 4$, is piecewise constant, and has this graph



Break the Laplace transform integral from zero to infinity into the sum of integrals over four subintervals (two of the integrals will be zero), in order to compute the Laplace transform of $f(t)$.

Use Laplace transform table, linearity (and useful trig identities) to compute Laplace transforms and inverse Laplace transforms.

13, 17, 19, 23, 29

w11.3a) Use the Laplace transform table in the front cover of the text to compute the Laplace transform $F(s)$ of

$$f(t) = 2 e^{-5t} \sin(2t) - 5 e^{3t} (2 - t).$$

b) Check your answer with technology. (Maple commands are shown at the end of this document, but you can use whatever software is most convenient.)

w11.4a) Use the Laplace transform table to compute the inverse Laplace transform $g(t)$ of

$$G(s) = \frac{3}{(s+2)^2} - \frac{3}{s^2+4}.$$

b) Check your answer with technology. (See below for Maple commands.)

10.2: Transforming and solving initial value problems via Laplace transforms;
3, 7, 9, 19.

w11.5a) Use Laplace transforms to solve the initial value problem for $x(t)$:

$$\begin{aligned}x''(t) + 5x'(t) + 6x(t) &= 0 \\x(0) &= x_0 \\x'(0) &= v_0\end{aligned}$$

b) Check your answer with technology.

10.2-10.3: Laplace transform table entries; partial fractions to simplify $F(s)$; the translation theorem with completing the square, to identify inverse Laplace transforms; applying these and other techniques to initial value problems.

10.2: **20**

10.3: 3, 7, 9, 17, **20, 30, 32, 34.**

w11.6) Practice completing the square (and "completing the linear") to find the inverse Laplace transform

$$f(t) \text{ for } F(s) = \frac{3s + 4}{s^2 + 4s + 29} .$$

w11.7) With access to a Laplace transform table it is possible to very quickly recover the general solutions to key mechanical oscillation problems. Do this for

a) undamped forced oscillation, $\omega \neq \omega_0$:

$$\begin{aligned}x''(t) + \omega_0^2 x(t) &= \frac{F_0}{m} \cos(\omega t) \\x(0) &= x_0 \\x'(0) &= v_0\end{aligned}$$

b) undamped forced oscillation, $\omega = \omega_0$:

$$\begin{aligned}x''(t) + \omega_0^2 x(t) &= \frac{F_0}{m} \cos(\omega_0 t) \\x(0) &= x_0 \\x'(0) &= v_0\end{aligned}$$

Notes: Here are Maple commands to check partial fractions and to compute Laplace transforms and inverse Laplace transforms:

> `with(intrans) : # to see the integral transform list in this library replace : with ;`

> `f1 := t -> t * exp(3 * t) * cos(4 * t);`

`laplace(f1(t), t, s); # for more info on this command use help windows`

$$f1 := t \rightarrow t e^{3t} \cos(4t)$$

$$\frac{s^2 - 6s - 7}{((s - 3)^2 + 16)^2}$$

(1)

```
> FI := s -> (s^2 - 6*s - 7) / ((s - 3)^2 + 16)^2;  
invlaplace(FI(s), s, t);
```

$$FI := s \rightarrow \frac{s^2 - 6s - 7}{((s - 3)^2 + 16)^2}$$
$$t e^{3t} \cos(4t)$$

(2)

```
> convert(FI(s), parfrac, s);
```

$$\frac{1}{s^2 - 6s + 25} - \frac{32}{(s^2 - 6s + 25)^2}$$

(3)

```
>
```