Name ____

Student I.D. ___________________________________

Math 2250-4
Exam 2
March 29, 2012

Please show all work for full credit. This exam is closed book and closed note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin. **Good Luck!**

	Score	POSSIBLE
1		20
2		25
3		20
4		20
5		15
TOTAL		100

1) Are $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix}$ a basis for $\mathbb{R}^3$? If they are, explain why. If they aren't a basis for $\mathbb{R}^3$, determine what sort of subspace of $\mathbb{R}^3$ they do span, and find a basis for that subspace.

(20 points)

2) Consider the following mechanical oscillation (or electrical circuit) differential equations. In each case answer the following questions:

(i) What is the undetermined coefficients "guess" for the particular solution $x_p(t)$? (Do NOT try to find the precise particular solution, just its form.)
(ii) What is the complete general solution to the non-homogeneous differential equation, including your undetermined coefficients particular solution?
(ii) What physical phenomenon will be exhibited by the general solutions to this differential equation?

2a) $x''(t) + 5\,x(t) = 3\cos(2\,t)$.

(8 points)

2b) $x''(t) + 4\,x(t) = 3\cos(2\,t)$

(8 points)

2c) $x''(t) + 0.2 \cdot x'(t) + 4.01\,x(t) = 3\cos(2\,t)$.

(9 points)

3) Use Chapter 5 techniques to solve the initial value problem

$$x''(t) + 9\,x(t) = 10\cos(2\,t)$$
$$x(0) = 2$$
$$x'(0) = 1$$

(20 points)

4) Re-solve the initial value problem in (3) using Laplace transform.

$$x''(t) + 9\,x(t) = 10\cos(2\,t)$$
$$x(0) = 2$$
$$x'(0) = 1$$

(20 points)

5a) Set up the partial fractions decomposition for

$$F(s) = \frac{4s + 10}{(s^2 + 4s + 13)(s + 2)^2 (s^2 + 16)^2} .$$

Do not try to find the actual values of the partial fraction coefficients.

(7 points)

5b) Use the Laplace transform table to find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$ for the $F(s)$ is part (a), in terms of the (unknown) partial fraction coefficients.

(8 points)

Table of Laplace Transforms

This table summarizes the general properties of Laplace transforms and the Laplace transforms of particular functions derived in Chapter 10.

Function	Transform	Function	Transform
$f(t)$	$F(s)$	e^{at}	$\dfrac{1}{s-a}$
$af(t)+bg(t)$	$aF(s)+bG(s)$	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
$f'(t)$	$sF(s)-f(0)$	$\cos kt$	$\dfrac{s}{s^2+k^2}$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$	$\sin kt$	$\dfrac{k}{s^2+k^2}$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$	$\cosh kt$	$\dfrac{s}{s^2-k^2}$
$\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$	$\sinh kt$	$\dfrac{k}{s^2-k^2}$
$e^{at}f(t)$	$F(s-a)$	$e^{at}\cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
$u(t-a)f(t-a)$	$e^{-as}F(s)$	$e^{at}\sin kt$	$\dfrac{k}{(s-a)^2+k^2}$
$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$	$\dfrac{1}{2k^3}(\sin kt - kt\cos kt)$	$\dfrac{1}{(s^2+k^2)^2}$
$tf(t)$	$-F'(s)$	$\dfrac{t}{2k}\sin kt$	$\dfrac{s}{(s^2+k^2)^2}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$\dfrac{1}{2k}(\sin kt + kt\cos kt)$	$\dfrac{s^2}{(s^2+k^2)^2}$
$\dfrac{f(t)}{t}$	$\int_s^\infty F(\sigma)\,d\sigma$	$u(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t)$, period p	$\dfrac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt$	$\delta(t-a)$	e^{-as}
1	$\dfrac{1}{s}$	$(-1)^{[\![t/a]\!]}$ (square wave)	$\dfrac{1}{s}\tanh\dfrac{as}{2}$
t	$\dfrac{1}{s^2}$	$\left[\!\left[\dfrac{t}{a}\right]\!\right]$ (staircase)	$\dfrac{e^{-as}}{s(1-e^{-as})}$
t^n	$\dfrac{n!}{s^{n+1}}$		
$\dfrac{1}{\sqrt{\pi t}}$	$\dfrac{1}{\sqrt{s}}$		
t^a	$\dfrac{\Gamma(a+1)}{s^{a+1}}$		