Math 2250-4
Tuesday 3/28 : Finish Monday notes & examples

Table of Laplace Transforms

This table summarizes the general properties of Laplace transforms and the Laplace transforms of particular functions derived in Chapter 10.

What you'll need for exam (10.1–10.3):

Function	Transform	Function	Transform
$f(t)$	$F(s)$	e^{at}	$\frac{1}{s-a}$
$af(t)+bg(t)$	$aF(s)+bG(s)$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$f'(t)$	$sF(s)-f(0)$	$\cos kt$	$\frac{s}{s^2+k^2}$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$	$\sin kt$	$\frac{k}{s^2+k^2}$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$	$\cosh kt$	$\frac{s}{s^2-k^2}$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$	$\sinh kt$	$\frac{k}{s^2-k^2}$
$e^{at}f(t)$	$F(s-a)$	$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
~~$u(t-a)f(t-a)$~~	~~$e^{-as}F(s)$~~	$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$
~~$\int_0^t f(\tau)g(t-\tau)\,d\tau$~~	~~$F(s)G(s)$~~	$\frac{1}{2k^3}(\sin kt - kt\cos kt)$	$\frac{1}{(s^2+k^2)^2}$
$tf(t)$	$-F'(s)$	$\frac{t}{2k}\sin kt$	$\frac{s}{(s^2+k^2)^2}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$\frac{1}{2k}(\sin kt + kt\cos kt)$	$\frac{s^2}{(s^2+k^2)^2}$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)\,d\sigma$	~~$u(t-a)$~~	~~$\frac{e^{-as}}{s}$~~
~~$f(t)$, period p~~	~~$\frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt$~~	~~$\delta(t-a)$~~	~~e^{-as}~~
1	$\frac{1}{s}$	~~$(-1)^{[\![t/a]\!]}$ (square wave)~~	~~$\frac{1}{s}\tanh\frac{as}{2}$~~
t	$\frac{1}{s^2}$	~~$[\![\frac{t}{a}]\!]$ (staircase)~~	~~$\frac{e^{-as}}{s(1-e^{-as})}$~~
t^n	$\frac{n!}{s^{n+1}}$		
~~$\frac{1}{\sqrt{\pi t}}$~~	~~$\frac{1}{\sqrt{s}}$~~		
~~t^a~~	~~$\frac{\Gamma(a+1)}{s^{a+1}}$~~		

Review Sheet
Math 2250–4, March 2011

Our exam covers chapters 4, 5, EP3.7, and 10.1–10.3 of the text. Only scientific calculators will be allowed on the exam. There is a practice session on Tuesday evening March 29, 6–7:30 p.m., in LCB 215 (next to our classroom). We will go over a practice exam, which is posted on our exam page. (As usual, no practice exam can cover all possible topics.)

Chapter 4: (except 4.6)

At most 30% of the exam will deal directly with this material....but much of Chapter 5 uses these concepts, so much more than 30% of the exam will be related to chapter 4. (And, as far as matrix computations go, you should remember everything you learned in Chapter 3.)

Know Definitions:

(a) **Vector Space:** A collection of objects which can be added and scalar multiplied, so that the usual arithemetic properties (Page 240) hold. You do not need to memorize all eight of these properties. The key point is that not only is R^n a vector space, but also certain subsets of it are, and so are spaces made out of functions... because functions can be added and scalar multiplied (page 241.)

(b) **Subspace:** a subset of a vector space which is itself of vector space....to check whether a subset is actually a subspace you only have to show that sums and scalar multiples of subset elements are also in the subset (Theorem 1 page 241.) Examples of important subspaces are the solution space to a homogeneous matrix equation (page 243), the span of a collection of vectors (page 247), AND the solution space to a homogeneous linear differential equation (pages 294, 301).

(c) A **linear combination** of a set of vectors $\{v_1, v_2 \dots v_n\}$ is any expression $c_1v_1 + c_2v_2 + \dots + c_nv_n$ (page 246).

(d) The **span** of a set of vectors $\{v_1, v_2 \dots v_n\}$ is the collection of all possible linear combinations. (page 243)

(e) A collection $\{v_1, v_2 \dots v_n\}$ is **linearly dependent** if and only if some linear combinatation (with not all c_i's $= 0$) adds up to the zero vector. (This is a symmetric way of saying that some vector in the collection is in the span of the remaining ones.)

(f) A collection $\{v_1, v_2 \dots v_n\}$ is **linearly independent** if and only if the only linear combination of them which adds up to zero is the one in which all coefficients $c_i = 0$. In other words, $c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$ implies $c_1 = c_2 = \dots = c_n = 0$ (page 248).

(g) A **basis** for a vector space (or subspace) is a set of vectors $\{v_1, v_2 \dots v_n\}$ which spans the space and which is linearly independent. (page 253.)

(h) The **dimension** of a vector space is the number of elements in any basis.

Know Facts:

(a) If the dimension of a vector space is n, then no collection of fewer than n vectors can span and every collection with more than n elements is dependent.

(b) n vectors in R^n are a basis if and only if the square matrix in which they are the columns is non–singular. So you can use det or rref as a test for basis in this case.

(c) Basically all linear independence and span questions in R^n can be answered using rref. (see below.)

(d) You can toss dependent vectors out of a collection without shrinking the span. In this manner you can take a spanning set and turn it into a basis.

(e) You can add a vector not in the span of an independent collection and preserve independence. In this manner you can take an independent set and enlarge it into a basis.

Do computations:

(a) Be able to check whether vectors are independent or independent, e.g. problems page 252–253. (4.3) Know how to use rref to check for dependencies.

(b) Be able to find bases for the solution space to homogeneous equations, e.g. problems page 260 (4.4)

(c) Be able to find bases for matrix row spaces and column spaces pages 268–269 (4.5).

Chapter 5 and EP3.7 (circuits).

At least 50% of the exam will cover this material, and at least 30% of the exam will be from sections 5.4 and 5.5. (Answering questions from 5.4 and 5.5 almost always uses 5.1–5.3 material implicitly.)

5.1–5.3, 5.5 General theory:

Linear differential equations (page 301.)

principle of superposition (e.g. Theorem 1 page 301, also leads to the fact that the general solution y to the inhomogeneous equation is $y_p + y_H$, where y_p is a particular solution, and y_H is the general solution to the homogeneous equation. (Theorem 5 page 310.) Also leads to a method for getting particular solutions which are sums of particular solutions for pieces of the right hand side.

homogeneous $(L(y) = 0)$. Solution space is an n–dimensional vector space if L is an n^{th} order linear differential equation operator. Know how to find the general solution (or a basis), for the constant coefficient operator, using exponentials and the resulting **characteristic equation** and **Euler formula** if necessary (sections 5.1, 5.3 and problems). What to do with repeated roots. The Wronskian test for linear independence .

nonhomogeneous $(L(y) = f)$. Know how to find particular solutions by the method of undetermined coefficients. (Section 5.5 and problems) (But variation of parameters will not be on the exam.)

initial value problem, existence and uniqueness. Know how to solve initial value problems by finding a y_p, the general y_H, and then finding values of linear combination coefficients in y_H so that $y = y_p + y_H$ solves the IVP.

5.4, 5.6, EP3.7 Mechanical vibrations and forced oscillations; electrical circuit analog

unforced oscillations (i.e. solutions to the homogeneous DE):

undamped (simple harmonic motion)

going from $A\cdot\cos(\omega\cdot t) + B\cdot\sin(\omega\cdot t) = C\cdot\cos(\omega\cdot t - \alpha)$ and back. (The ABC triangle, amplitude and phase.)

derivation of spring equation from Newton's and Hooke's Laws.

using total energy (KE+PE) is constant in conservative systems, te decuce equation of motion (e.g. pendulum).

damped.

under–damped, over–damped, critically damped. Know how to recognize, and different forms of the solution.

forced oscillations:

undamped:

resonance, and when it arises. form of solution, as follows from general theory above.

beating, when ω is close to ω_0.

damped:

general solution is sum of **steady state periodic**, with **transient**. How to find each piece, and express the steady state periodic solution in amplitude– phase form.

practical resonance, will occur if damping is small and driving frequency is near natural frequency.

An RLC circuit corresponds to a mass–spring configuration, where L corresponds to mass, R to damping coefficient, and 1/C to spring constant, and the applied voltage corresponds to the force function. (Kirchoff's law says net voltage drop (PE) around and circuit loop is zero.)

Chapter 10:

At least 25% of the exam will deal directly with this material. The most efficient way for me to test it is to have you solve IVPs from chapter 5, using chapter 5 techniques as well as Laplace transform techniques. You will be responsible for 10.1–10.3 on the exam. (Sections 10.4–10.5 are yet to come.) You will be provided with the Laplace transform table on the front book cover. The one thing that table is missing is the definition of Laplace transform, which you should memorize (page 577).

Key skills:

Using the definition of Laplace transform to reproduce a table entry or to compute the transform of a function not
on the table.

Converting an initial value problem into an algebraic equation for the Laplace transform of the solution (section 10.2).

Using the table to convert in both directions between functions and their Laplace transforms. (covers 10.1–10.3 facts)

Setting up and using partial fractions (and possibly **completing the square**) to simplify Laplace transforms so that you can use the table to invert them. (section 10.3)