

Math 2250-4
Fri. March 11

Experiment/modeling day!

Hw for Fri 3/18

(1)

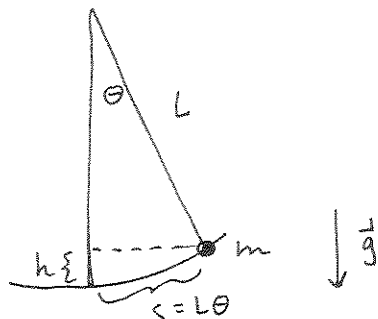
E.P. 3.7 (circuits)

(2, 3, 4, 7, 11, 20)

5.5 (3) 4 (12) 13, (19, 34, 37, 43, 49) 50 (52)

5.6 (3, 5, 7, 14, 16, 21, 22)

pendulum



conservative system $KE + PE = \text{const.}$

$$\frac{1}{2}mv^2 + mgh = \text{const}$$

$$s = L\theta$$

$$v = \frac{ds}{dt} = L\theta'(t)$$

$$h = L - L\cos\theta = L(1 - \cos\theta)$$

so, $\frac{1}{2}mL^2(\theta'(t))^2 + mgL(1 - \cos(\theta(t))) = \text{const}$

$$D_t: mL^2\theta'\theta'' + mgL(\sin\theta)\theta' = 0$$

$$\underbrace{mL\theta'}_{\neq 0 \text{ except at isolated times}} (L\theta'' + g\sin\theta) = 0$$

$\neq 0$ except
at isolated
times

$\sim$ deduce eqn of motion is

linearize $\boxed{\theta'' + \frac{g}{L}\sin\theta = 0}$

$$\boxed{\theta'' + \frac{g}{L}\theta = 0}$$

$$\omega_0 = \sqrt{\frac{g}{L}}$$

$$\theta(t) = C\cos(\omega_0 t - \alpha)$$

$\searrow$ non-linear DE

but $\sin\theta = \theta - \frac{\theta^3}{3!} + \dots$

$\sin\theta \approx \theta$ θ small

is excellent approx

(alternating series test)

Prediction:

Here is my measurement of L , which we'll check again in class. It assumes the effective length of the pendulum uses the the distance to the ball's center of mass, from the top. (And, where exactly is the top?!)

```
> L:=1.529;
g:=9.806;
omega:=sqrt(g/L); #radians per second
f:=evalf(omega/(2*Pi)); #cycles per second
T:=1/f; #seconds per cycle
```

$$L := 1.529$$

$$g := 9.806$$

$$\omega := 2.532$$

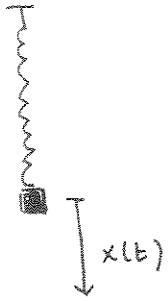
$$f := 0.4030$$

$$T := 2.481$$

Experiment:

error sources?

mass-spring



prediction: (we'll check!)

Mass-spring:

How Hookes constant:

```
> 91.7-76.2; #add 50g and measure displacement
15.5
```

```
> m:=.1; #the mass is 100g = 0.1 kg
```

```
m := 0.1
```

My measurement for k, using a 50g mass and measuring the displacement in meters

```
> k*.155=.05*g;
```

```
0.155 k = 0.4903
```

```
> k:=.05*g/.155;
```

```
k := 3.163
```

```
> omega:=sqrt(k/m);#radians per second
```

```
f:=omega/evalf(2*Pi); #cycles per second
```

```
T:=1/f; #seconds per cycle
```

```
ω := 5.624
```

```
f := 0.8950
```

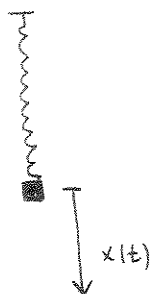
```
T := 1.117
```

experiment:

error sources?

(there's a pretty significant
one)

Improved spring model (account for spring mass)



normalize $KE + PE = 0$ for spring hanging at rest. (in gravity!)

then for system in motion

$$KE + PE = \frac{1}{2} m (\dot{x})^2 + KE_{\text{spring}} + \underbrace{\frac{1}{2} k x^2}_{\text{PE of system from equilibrium.}}$$

model: the top of the spring isn't moving. the bottom is moving with velocity $\dot{x}(t)$.

Assume the velocity of a piece of spring a fraction μ of the way ($0 \leq \mu \leq 1$) from top to bottom is $\mu \dot{x}(t)$.

so $\text{speed}^2 = \mu^2 (\dot{x})^2$, for that piece. Thus

$$KE = \int_0^1 \underbrace{\frac{1}{2} \mu^2 (\dot{x}(t))^2}_{v^2} \underbrace{m_s d\mu}_{\text{mass}} = \frac{1}{6} m_s (\dot{x}(t))^2$$

So for $M := m + \frac{1}{3} m_s$,

$$KE + PE = \frac{1}{2} M (\dot{x})^2 + \frac{1}{2} k x^2 \equiv \text{const}$$

$$D_t: \quad M \ddot{x} + k x \equiv 0$$

$$\ddot{x} (\underbrace{M \ddot{x} + k x}_{=0}) \equiv 0$$

$$\omega_0 = \sqrt{\frac{k}{M}}$$

Does this improve our prediction?

```
> ms:=.010; #spring has mass 11g
M:=m+ms/3; #"effective mass"
M:=0.1033
```

```
> omegal:=sqrt(k/M); #new angular freq est.
f1:=omegal/evalf(2*Pi); #new freq est.
T1:=1/f1; #new period est
```

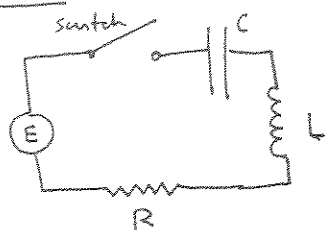
$\omega_l := 5.534$

$f_l := 0.8806$

$T_l := 1.136$

>

EP 3.7 (Electrical Circuits)



Voltage E volts (called Electromotive force, but really is energy/unit charge)

circuit elt	Voltage drop	units for component
Inductor	$L \frac{dI}{dt}$	L Henries H
Resistor	$R I(t)$	R ohms Ω
Capacitor	$\frac{1}{C} Q(t)$	C Farads F

$Q(t)$ = net charge on Capacitor (~~C~~ Coulombs)

$I(t) = Q'(t)$ = current (Amperes)
(moving around loop).

(1 Volt = 1 Joule/Coulomb!)

Exercise 1: Set up an IVP for a circuit in which

$$R = 16 \Omega$$

$$L = 2 H$$

$$C = .02 F$$

$$E(t) = 100 V$$

$$I(0) = 0$$

$$Q(0) = 0$$

Kirchoff's Law

the sum of the voltage drops around a circuit loop equals the applied voltage E .

(this says that a test particle traversing a closed loop returns with the same potential energy level it started at.)



$$L Q''(t) + R Q'(t) + \frac{1}{C} Q(t) = E(t)$$

D_t :

$$L I''(t) + R I'(t) + \frac{1}{C} I(t) = E'(t)$$

mathematically identical to

$$m x'' + c x' + k x = F(t)$$

$\downarrow$
 L

$\downarrow$
 R

$\downarrow$
 $1/C$

IVP for $Q(t)$:

IVP for $I(t)$:

5

Named units derived from SI base units

Name	Symbol	Quantity	Expression in terms of other units	Expression in terms of SI base units
becquerel	Bq	radioactivity (decays per unit time)	1/s	s ⁻¹
Celsius	°C	Celsius temperature	K − 273.15	K − 273.15
coulomb	C	electric charge or quantity of electricity	s·A	s·A
farad	F	electric capacitance	C/V	m ⁻² ·kg ⁻¹ ·s ⁴ ·A ²
gray	Gy	absorbed dose (of ionizing radiation)	J/kg	m ² ·s ⁻²
henry	H	inductance	V·s/A = Wb/A	m ² ·kg·s ⁻² ·A ⁻²
hertz	Hz	frequency	1/s	s ⁻¹
joule	J	energy, work, heat	N·m = C·V = W·s	m ² ·kg·s ⁻²
katal	kat	catalytic activity	mol/s	s ⁻¹ ·mol
lumen	lm	luminous flux	lx·m ²	cd·sr
lux	lx	illuminance	lm/m ²	m ⁻² ·cd·sr
newton	N	force, weight	kg·m/s ²	kg·m·s ⁻²
ohm	Ω	electric resistance, impedance, reactance	V/A	m ² ·kg·s ⁻³ ·A ⁻²
pascal	Pa	pressure, stress	N/m ²	m ⁻¹ ·kg·s ⁻²
radian	rad	angle	m·m ⁻¹	dimensionless
siemens	S	electrical conductance	1/Ω = A/V	m ⁻² ·kg ⁻¹ ·s ³ ·A ²
sievert	Sv	equivalent dose (of ionizing radiation)	J/kg	m ² ·s ⁻²
steradian	sr	solid angle	m ² ·m ⁻²	dimensionless
tesla	T	magnetic field strength, magnetic flux density	V·s/m ² = Wb/m ² = N/(A·m)	kg·s ⁻² ·A ⁻¹
volt	V	voltage, electrical potential difference, electromotive force	W/A = J/C	m ² ·kg·s ⁻³ ·A ⁻¹
watt	W	power, radiant flux	J/s = V·A	m ² ·kg·s ⁻³
weber	Wb	magnetic flux	J/A	m ² ·kg·s ⁻² ·A ⁻¹

note: 1 Coulomb = charge in $\approx 6.24 \times 10^{18}$ electrons

How Oscillators Work

by Marshall Brain

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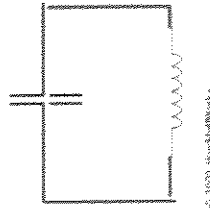
Inside this Article

1. Introduction to How Oscillators Work
2. Oscillation Basics
3. Oscillator Circuits
4. Resonators
5. Lots More Information
6. See all Solid State Electronics articles

Oscillator Circuits

Energy needs to move back and forth from one form to another for an oscillator to work. You can make a very simple oscillator by connecting a [capacitor](#) and an [inductor](#) together. If you've read [How Capacitors Work](#) and [How Inductors Work](#), you know that both capacitors and inductors **store energy**. A capacitor stores energy in the form of an electrostatic field, while an inductor uses a magnetic field.

Imagine the following circuit:



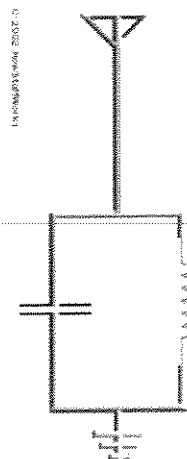
If you charge up the capacitor with a [battery](#) and then insert the inductor into the circuit, here's what will happen:

- The capacitor will start to discharge through the inductor. As it does, the inductor will create a magnetic field.
- Once the capacitor discharges, the inductor will try to keep the current in the circuit moving, so it will charge up the other plate of the capacitor.
- Once the inductor's field collapses, the capacitor has been recharged (but with the opposite polarity), so it discharges again through the inductor.

This oscillation will continue until the circuit runs out of energy due to **resistance** in the wire. It will oscillate at a frequency that depends on the size of the inductor and the capacitor.

Resonators

In a simple crystal radio (see [How Radio Works](#) for details), a capacitor/inductor oscillator acts as the **tuner** for the radio. It is connected to an antenna and ground like this:



Thousands of **sine waves** from different radio stations hit the antenna. The capacitor and inductor want to resonate at one particular frequency. The sine wave that matches that particular frequency will get **amplified** by the resonator, and all of the other frequencies will be ignored.

In a radio, either the capacitor or the inductor in the resonator is **adjustable**. When you turn the tuner knob on the radio, you are adjusting, for example, a variable capacitor. Varying the capacitor changes the resonant frequency of the resonator and therefore changes the frequency of the sine wave that the resonator amplifies. This is how you "tune in" different stations on the radio!

How Inductors Work

by Marshall Brain

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4. Inductor Application:



Chris Pollette added 19 quiz points:

2 Ultimate LEGO Quiz

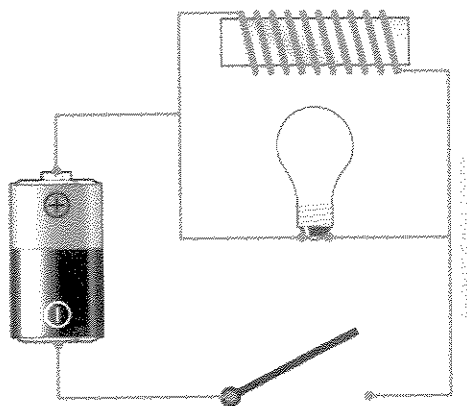
Electronics articles

Inductor Basics

In a circuit diagram, an inductor is shown like this:



To understand how an inductor can work in a circuit, this figure is helpful:



What you see here is a battery, a light bulb, a coil of wire around a piece of iron (yellow) and a switch. The coil of wire is an **inductor**. If you have read [How Electromagnets Work](#), you might recognize that the inductor is an electromagnet.

If you were to take the inductor out of this circuit, what you would have is a normal flashlight. You close the switch and the bulb lights up. With the inductor in the circuit as shown, the behavior is completely different.

The light bulb is a **resistor** (the resistance creates heat to make the filament in the bulb glow -- see [How Light Bulbs Work](#) for details). The wire in the coil has much lower resistance (it's just wire), so what you would expect when you turn on the switch is for the bulb to glow very dimly. Most of the current should follow the low-resistance path through the loop. What happens instead is that when you close the switch, the bulb burns brightly and then gets dimmer. When you open the switch, the bulb burns very brightly and then quickly goes out.

The reason for this strange behavior is the inductor. When current first starts flowing in the coil, the coil wants to build up a **magnetic field**. While the field is building, the coil inhibits the flow of current. Once the field is built, current can flow normally through the wire. When the switch gets opened, the magnetic field around the coil keeps current flowing in the coil until the field collapses.

This current keeps the bulb lit for a period of time even though the switch is open. In other words, an inductor can **store energy** in its magnetic field, and an inductor tends to resist any change in the amount of current flowing through it.

Think About Water...

One way to visualize the action of an inductor is to imagine a narrow channel with water flowing through it, and a heavy water wheel that has its

paddles dipping into the channel. Imagine that the water in the channel is not flowing initially.

Now you try to start the water flowing. The paddle wheel will tend to prevent the water from flowing until it has come up to speed with the water. If you then try to stop the flow of water in the channel, the spinning water wheel will try to keep the water moving until its speed of rotation slows back down to the speed of the water. An inductor is doing the same thing with the flow of electrons in a wire -- an **inductor resists a change in the flow of electrons**.