

Math 2250/2280  
First Maple Assignment **SOLUTIONS**  
January 28, 2011

Find, use and modify commands to do the following:

0) Begin by downloading this document and opening it from Maple. A shortcut is to use the "open URL" option in the "File" menu item. The URL's below give links to this document and to a list of commands you might find helpful.

<http://www.math.utah.edu/~korevaar/2250spring11/maplehw1.mw>  
<http://www.math.utah.edu/~korevaar/2250spring11/maplecommands.mw>

(The general way to find commands that will do what you want is to use Maple Help, look for the options on the left-hand menu, cut and paste from a related document, and/or use google.)

Now that you've opened this document from Maple, insert text and execute math commands as appropriate to create well-formatted and clear responses to the following questions.

*I'll insert italics text into the document for my answers, to differentiate from the original assignment. Also, I'll do all math in "[>]" execution fields and format the document so that even after removing output, executing the document results in the correct output being recreated. I'll start with a clean slate by using "restart". All commands are found in the commands file linked to above.*

*I've removed the output from this file (an option under the "edit" icon), but created the file in such a way that the correct output is regenerated by re-executing the entire document (!!! in the menu bar, or as another option under the "edit" icon).*

> restart :  
>

1) Use Maple to define the function  $F(x) = \cos(3x + 1) + e^{-x} + 1$ .

>  $F := x \rightarrow \cos(3 \cdot x + 1) + \exp(-x) + 1;$

#Use the exp function, as Maple does not recognize the  $e$  in  $e^x$  as being the number 2.718 ...

$$F := x \rightarrow \cos(3x + 1) + e^{-x} + 1 \quad (1)$$

2) Have maple compute the derivative of  $F(x)$  and then define this derivative function to be  $f(x)$ .

>  $\text{diff}(F(x), x);$

$$-3 \sin(3x + 1) - e^{-x} \quad (2)$$

>  $f := x \rightarrow -3 \sin(3x + 1) - e^{-x};$  # copied and pasted previous output  
 $f(2);$  #and now I can evaluate at points

$$f := x \rightarrow -3 \sin(3x + 1) - e^{-x} \\ -3 \sin(7) - e^{-2} \quad (3)$$

*Subtlety: If you want to directly say that  $f(x)$  is the derivative of  $F(x)$ , you will run into syntax problems if you try to evaluate at a point:*

>  $f := x \rightarrow \text{diff}(F(x), x);$

$f(x)$ ; #looks good  
 $f(1)$ ; #but this is bad

$$f := x \rightarrow \frac{d}{dx} F(x)$$

$$-3 \sin(3x + 1) - e^{-x}$$

Error, (in f) invalid input: diff received 1, which is not valid for its 2nd argument

What happened is that the subroutine first set  $x=1$ , then tried to differentiate with respect to the number 1, which is not possible. The way around this is to use the substitution command, if you don't want to copy and paste:

>  $f := x \rightarrow \text{subs}(t=x, \text{diff}(F(t), t))$ ; #first differentiate, then substitute  
 $f(x)$ ;  
 $f(1)$ ;

$$f := x \rightarrow \text{subs}\left(t=x, \frac{d}{dt} F(t)\right)$$

$$-3 \sin(3x + 1) - e^{-x}$$

$$-3 \sin(4) - e^{-1}$$

(4)

3) Have maple antidifferentiate  $f(x)$ . Note that Maple does not add an additive constant when it antidifferentiates, so that your result won't exactly equal  $F(x)$ .

>  $\text{int}(f(x), x)$ ; #differs from  $F(x)$  by

$$\cos(3x + 1) + e^{-x}$$

(5)

4) Use Maple to verify the Fundamental Theorems of Calculus for these examples, namely

$$\int_a^b f(x) \, dx = F(b) - F(a) \quad \text{and}$$

$$\frac{\partial}{\partial x} \left( \int_a^x f(r) \, dr \right) = f(x).$$

I'll compute each side of each equation, verify they're equal visually, and check by taking the differences:

>  $LHS1 := \text{int}(f(x), x=a..b)$ ;  
 $RHS1 := F(b) - F(a)$ ;  
 $LHS1 - RHS1$ ;

$$LHS1 := -\cos(3a + 1) - e^{-a} + \cos(3b + 1) + e^{-b}$$

$$RHS1 := -\cos(3a + 1) - e^{-a} + \cos(3b + 1) + e^{-b}$$

$$0$$

(6)

>  $LHS2 := \text{diff}(\text{int}(f(r), r=a..x), x)$ ;  
 $RHS2 := f(x)$ ;  
 $LHS2 - RHS2$ ;

$$LHS2 := -3 \sin(3x + 1) - e^{-x}$$

$$RHS2 := -3 \sin(3x + 1) - e^{-x}$$

$$0$$

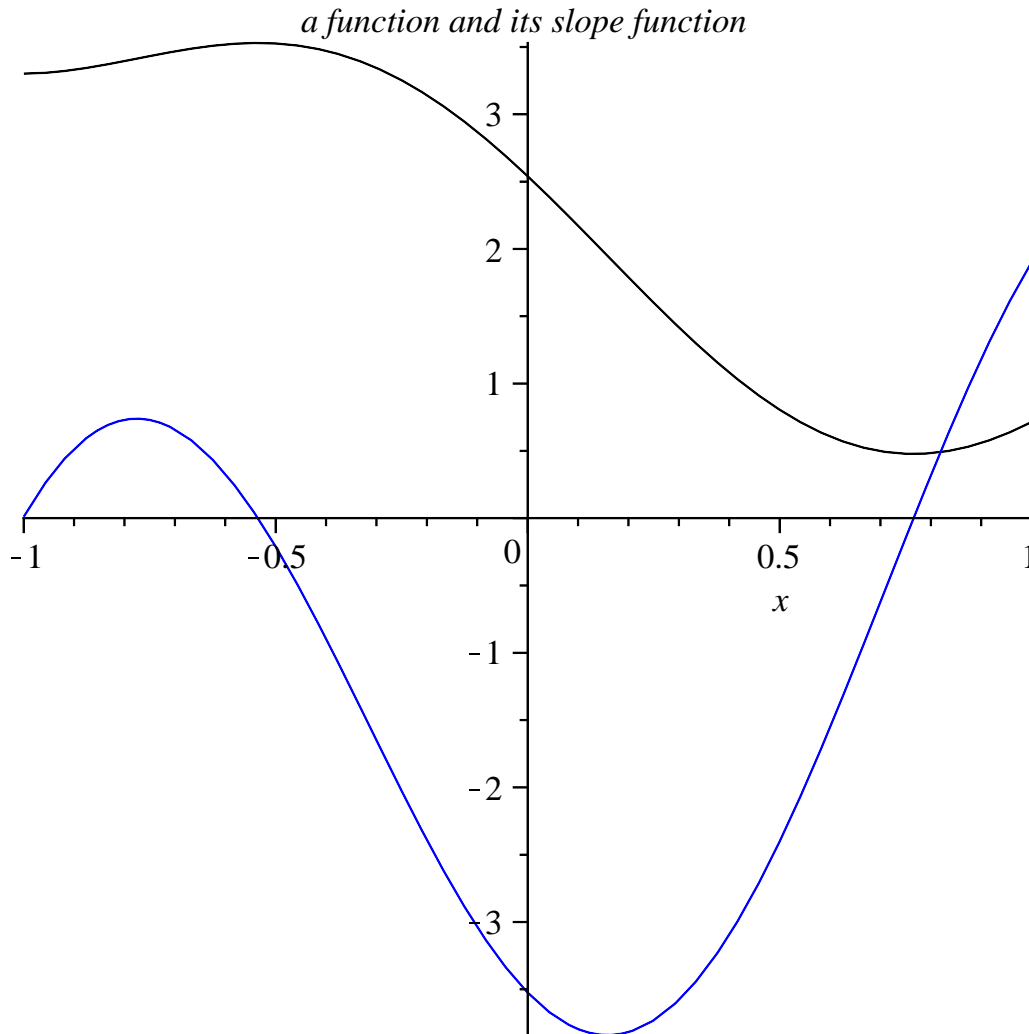
(7)

5) Create a single plot which contains the graphs of  $f(x)$  and  $F(x)$ , for  $-1 \leq x \leq 1$ . Explain how the two graphs are related.

```

> with(plots) : #plotting library
> plot1 := plot(f(x), x=-1..1, color=blue) :
  plot2 := plot(F(x), x=-1..1, color=black) :
  display( {plot1, plot2}, title = 'a function and its slope function');

```



*I resized the plot to make it smaller than it first appeared (just grab a corner with your mouse and move it). The blue (lower) curve is giving the slope of the black (upper curve). Since the horizontal scale is quite stretched relative to the vertical one, slopes are about 3 times greater than they appear to the eye. (Using the FTC you might also say that  $F(x)$  is the accumulation function for  $f(x)$ , up to an additive constant.)*

6) Define the 2-variable function  $g(x, y) = \sin(y) e^{0.5x}$ . Compute its  $x$  and  $y$  partial derivatives with Maple.

```

> g := (x, y) → sin(y) · exp(0.5 · x);

```

$$g := (x, y) \rightarrow \sin(y) e^{0.5x} \quad (8)$$

```

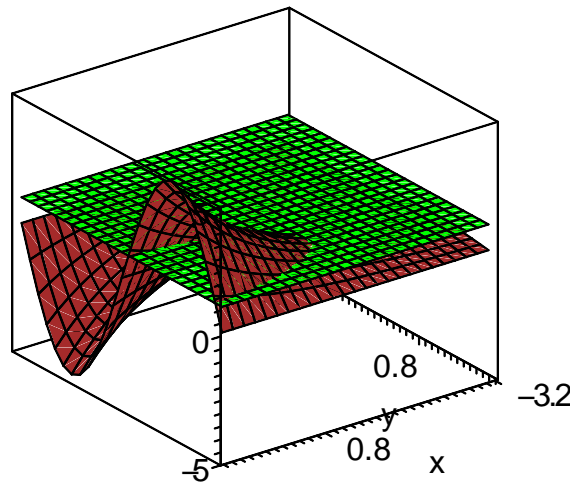
> diff(g(x, y), x); #x-partial
  diff(g(x, y), y); #y-partial

```

$$\begin{aligned} &0.5 \sin(y) e^{0.5x} \\ &\cos(y) e^{0.5x} \end{aligned} \quad (9)$$

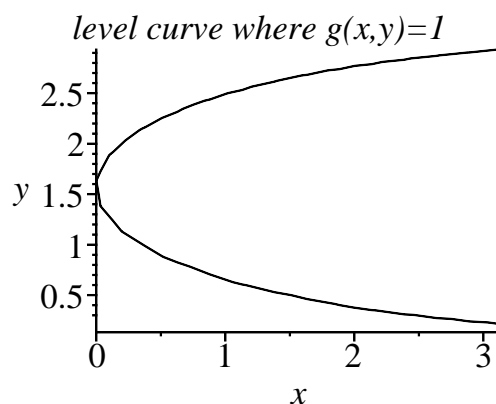
7) Create a single display containing the graph  $z = g(x, y)$ , and the horizontal plane  $z = 1$ . Use the square domain  $-\pi < x < \pi$ ,  $-\pi < y < \pi$ . Include boxed axes, orient your display, and pick other options to make the picture as informative as possible.

```
> plot3 := plot3d(g(x, y), x = -pi..pi, y = -pi..pi, color = brown) :
    #the number 3.14.159... is represented by (capital) Pi. Lower case pi is just a letter to
    Maple
plot4 := plot3d(1, x = -pi..pi, y = -pi..pi, color = green) :
display({plot3, plot4}, axes = boxed);
    #I re-oriented the plots with my mouse to make the picture more clear.
```



8) Plot the part of the curve in the  $x$ - $y$  plane which is given implicitly by the equation  $1 = \sin(y) e^{0.5x}$ , in the same  $x$ - $y$  square as (7). Explain what this curve has to do with your 3-d display in (7).

```
> implicitplot(g(x, y) = 1, x = -pi..pi, y = -pi..pi, color = black, title
    = 'level curve where g(x,y)=1');
```



The curve above is the part of the level curve where  $g(x, y) = 1$ , inside the given coordinate square, and is congruent to the curve of intersection between the horizontal plane at height 1 and the graph of  $g(x, y)$  above the same square, and shown in (7).