Math 2250-4
Fri 1/14 §1.3-1.4
Monday is Martin Luther King Day holiday!

Today: separable DE's theory & examples (§1.4)
used to illustrate & understand general existence-uniqueness thm for first order DE IVP's.

Tuesday: more applications of separable DE's. (We already did Newton's Law of cooling & exponential growth)

- The algorithm for separable DE's, why it works, and assumptions:

DE separable means:

$$\frac{dy}{dx} = f(x)\phi(y) = \frac{f(x)}{g(y)} \qquad \left(\text{as long as } \phi(y) \neq 0;\ g(y) = \frac{1}{\phi(y)}\right)$$

legal math sol'n

$$g(y(x))\,y'(x) = f(x)$$

If $\int g(y)\,dy = G(y)$, $\int f(x)\,dx = F(x)$
are antiderivs of $g(y)$ & $f(x)$

$$\frac{d}{dx} G(y(x)) = \frac{d}{dx} F(x) \qquad \text{on an } x\text{-interval}$$

$$\Rightarrow \quad G(y(x)) = F(x) + C$$

$$G(y) = F(x) + C$$

expresses $y(x)$ implicitly as a fcn of x.
you may be able to use algebra to get an explicit formula for y as a fcn of x

algorithm

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$G(y) = F(x) + C$$

same answer!

HW for Fri 1/21:
1.3 3, 8, 11, 12, 13, 14, 21, 24
for 8, 24 use dfield (google it.)
1.4 3, 4, 9, 12, 14, 19, 20, 22, 40, 45, 49, 54
1.5 1, 7, 8, 13, 20, 33, 36, 38, 41

Also go through Maple Intro notes on our course home page. Try Maple. Attend an introductory session next week, if you so desire.

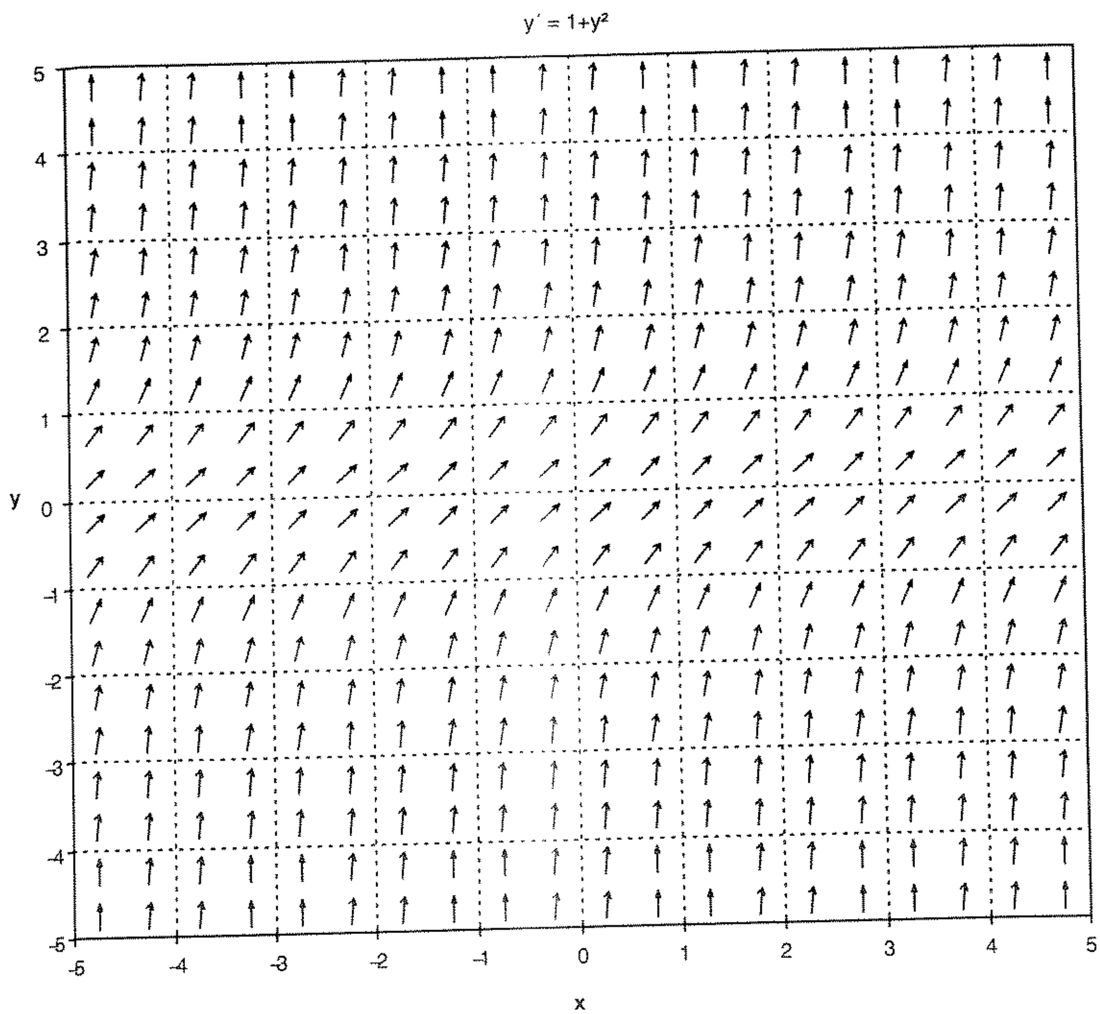
issues: if $\phi(y_0) = 0$ then algorithm fails ($g(y_0)$ not defined)
but you do get constant solutions
$$y(x) = y_0 \quad \forall x.$$
(since for this const. soltn, $\frac{dy}{dx} = 0$
$f(x)\phi(y_0) = 0$.

Exercise 1a) Use sep. vars to solve

$$\frac{dy}{dx} = 1 + y^2$$

1b) sketch some soltn graphs onto slope field
(this is actually a quick way to describe the slope field if you know the soltns)

1c) explain why each IVP has a soltn, but this soltn does not exist for all x



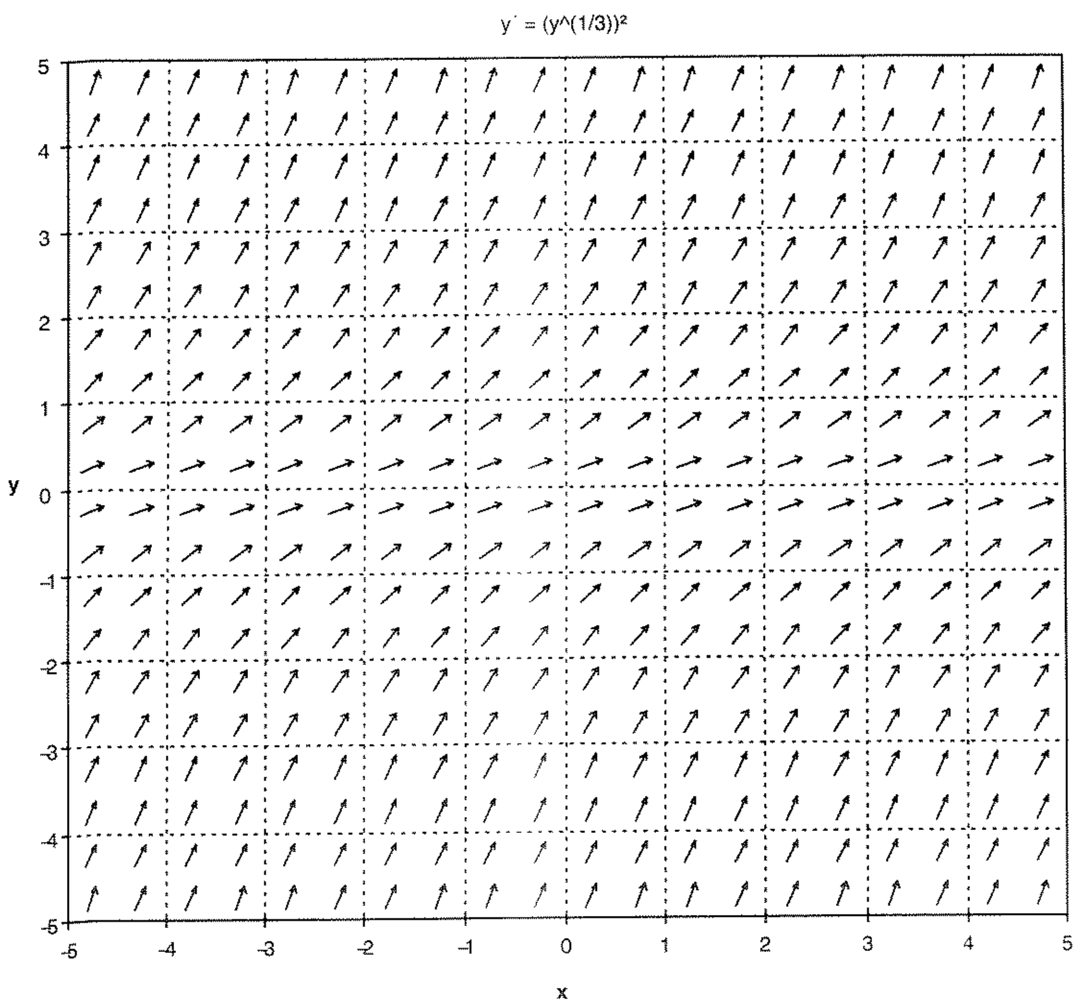
Exercise 2

2a) Solve $\begin{cases} \frac{dy}{dx} = y^{2/3} \\ y(0) = 0 \end{cases}$

use separation of variables

2b) But there are actually a lot more soltns to this IVP! (Ones you don't get from sep. var's are called singular soltns, see page 1.).

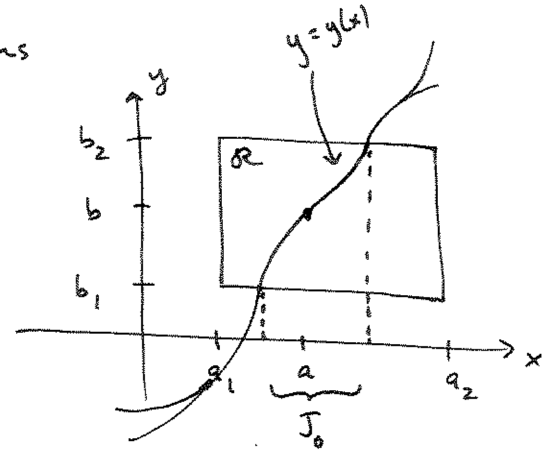
2c) Sketch some of these onto the slope field.



§1.3 : slope fields; existence and uniqueness for IVP's.

One reason we've been drawing slope fields is to motivate an existence and uniqueness theorem for 1^{st} order DE initial value problems

<u>Theorem 1</u> Existence and uniqueness of solutions
$\hookrightarrow \exists$ $\hookrightarrow !$



Consider

$$\text{IVP} \begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

Let the point (a,b) be interior to a coordinate rectangle R $\left(\begin{matrix} a_1 \le x \le a_2 \\ b_1 \le y \le b_2 \end{matrix}\right)$

in the x-y plane

- Let $f(x,y)$ be continuous in R
  Then
  $\exists$ sol'n to IVP on some interval J containing a in its interior
- If $\frac{\partial f}{\partial y}$ exists and is continuous in R then this solution is unique on any subinterval J_0 such that the solution graph lies inside R

proof: Appendix A
intuition: "follow the slope field" to get solt'ns.
$\frac{\partial f}{\partial y}$ continuous turns out to guarantee that multiple solutions can't peel off!

<u>Exercise 3</u> : Discuss $\exists !$ theorem in previous examples. (where we used Calculus and already knew $\exists !$)
We'll do more interesting examples,
for example in which uniqueness fails, on ~~We~~ Friday.

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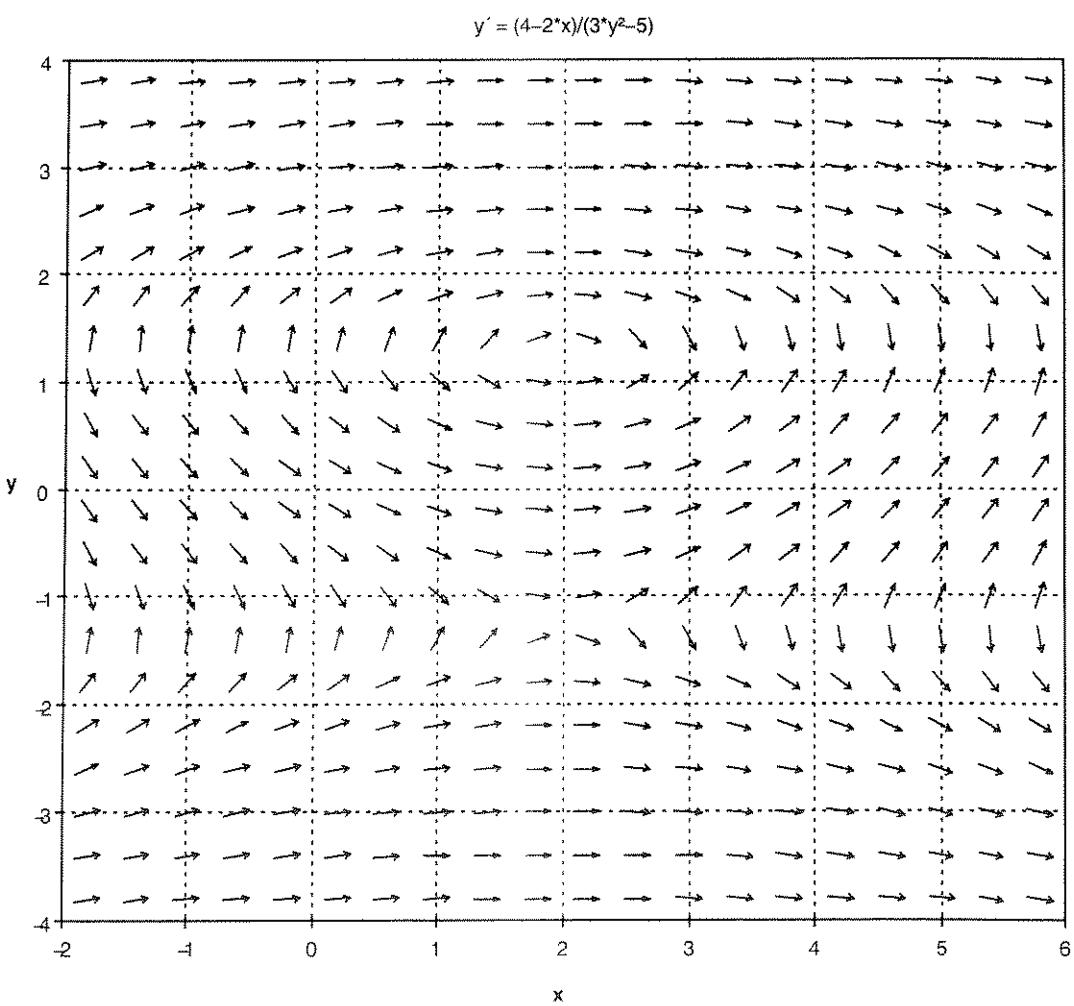
Use separation of vars to solve

a) $\frac{dy}{dx} = \frac{4-2x}{3y^2-5}$

4b) for what IVP's would the existence-uniqueness theorem fail for this DE? i.e. what (x_0, y_0)'s.

4c) Explain (4b) in terms of the slope field.

Note: since we can solve separable DE's explicitly (being careful about singular soltns), we don't really need the existence-uniqueness ∃ ! theorem in this case. But these examples provide good illustrations



If time, dicuss $\exists!$ for

$$\begin{cases} \frac{dy}{dx} = 6x(y-1)^{2/3} \\ y(x_0) = x_0 \end{cases}$$