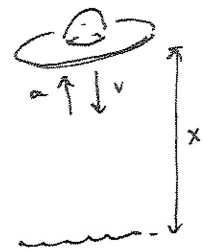
Math 2250-4
Wed. Jan 12   §1.2-1.3 cont'd.

§1.2 1^{st} order DE's you can solve by antidifferentiation
new applications!   Notice that setting up the model is a significant part of the problem.

<u>example 1</u> (is example 2 in text, p. 13-14)

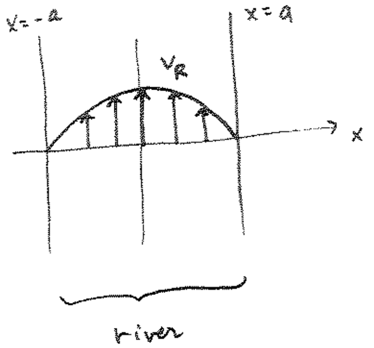
lunar lander



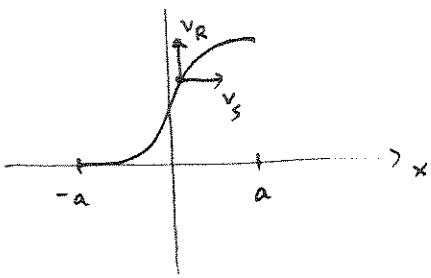
velocity to moon is 450 m/s.
retro-rockets provide net decelleration of
2.5 m/s^2.
At what height to fire rockets to get soft landing, i.e. $v=0$ exactly when height $x=0$?

ans: 40,500 m

<u>example 2</u> (actually example 4, the swimmer's problem, page 16)



$v_R = v_0\left(1 - \frac{x^2}{a^2}\right)$ is velocity profile of river water



swimmer swims due east with constant velocity v_s.
she starts at $(-a, 0)$.

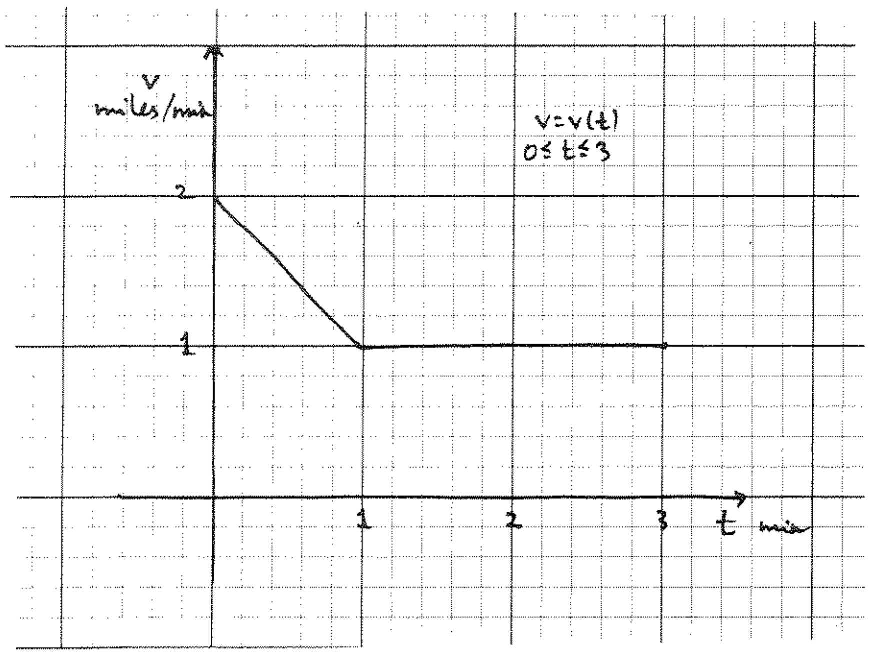
find the function $y = f(x)$ describing whose graph is her route!!

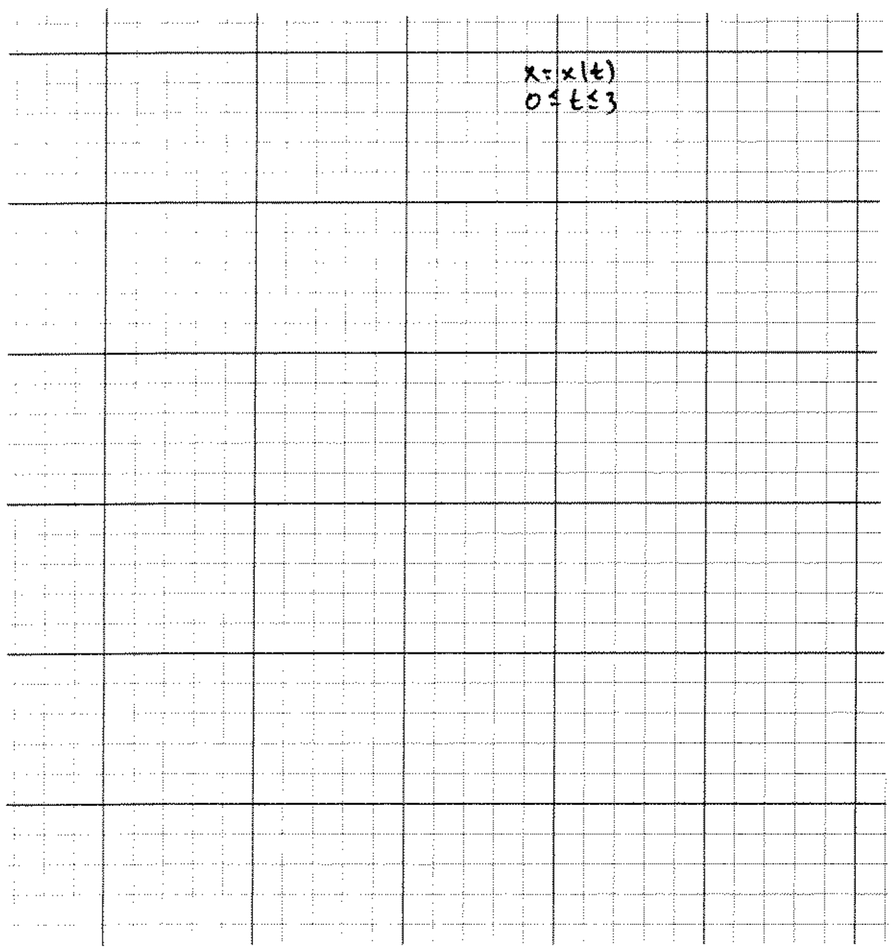
then, if river is 1 mile wide
if $v_s = 3$ mi/h (this is a very good swimmer)
$v_0 = 9$ mi/h (pretty fast river)

where does swimmer land on opposite shore?

ans: $y = 2$.

example 3 The velocity $v = \frac{dx}{dt}$ of a car traveling along the x-axis is shown in the following graph. The car starts at $x_0 = 0$.
Sketch the graph of its location at time t (and, find a formula for $x(t)$)!
How far did the car travel, $0 \le t \le 3$ minutes?





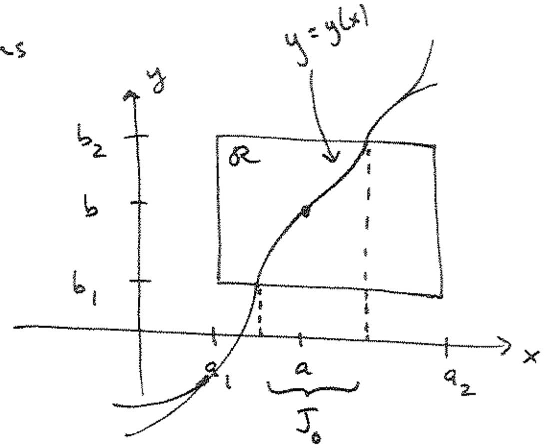
4b) extra: could you find exact formula for $x(t)$ i.e.

$$x(t) = \begin{cases} ? & 0 \le t \le 1 \\ ?? & t \ge 1 \end{cases}$$

§1.3 : slope fields; existence and uniqueness for IVP's.

One reason we've been drawing slope fields is to motivate an existence and uniqueness theorem for 1^{st} order DE initial value problems

Theorem 1 Existence and uniqueness of solutions
$\hookrightarrow \exists$ $\qquad$ $\hookrightarrow !$



Consider

$$\text{IVP} \begin{cases} \dfrac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

Let the point (a,b) be interior to a coordinate rectangle $\mathcal{R}$ $\left(\begin{matrix} a_1 \le x \le a_2 \\ b_1 \le y \le b_2 \end{matrix} \right)$

in the x-y plane

- Let $f(x,y)$ be continuous in $\mathcal{R}$
  Then
  $\exists$ sol'n to IVP on some interval J containing a in its interior
- If $\frac{\partial f}{\partial y}$ exists and is continuous in $\mathcal{R}$ then this solution is unique on any subinterval J_0 such that the solution graph lies inside $\mathcal{R}$

proof: Appendix A
intuition: "follow the slope field" to get soltns.
$\frac{\partial f}{\partial y}$ continuous turns out to guarantee that multiple solutions can't peel off!

<u>Exercise 4</u> : Discuss $\exists!$ theorem in previous examples. (where we used Calculus and already knew $\exists!$)
We'll do more interesting examples,
for example in which uniqueness fails, on Friday.