Math 2250-4
Monday Jan 10
MTWF 10:45-11:35 LCB 219

Due Jan 14 (this Fri)
1.1 3, 6 15 16 19 27, 31, 34, 35, 36
1.2 6 7, 13, 18 19, 20 25 29, 33, 34, 39
circled: hand in
uncircled: optional

- what is a differential equation?
  - n^{th} order DE: $F(x, y, y', y'', \ldots y^{(n)}) = 0$

    i.e. an equation involving a variable "x" and an (unknown) function $y(x)$ & its first n derivatives

- where do differential eqtns come from?
  - mathematical models, often of continuous dynamical systems in which the variable "x" is actually time "t".

- goal:
  - understand the "solution function(s)" $y(x)$, i.e. the functions for which the differential equation is true.

Examples:

(1) In calculus you have seen

$$\frac{dP}{dt} = kP \qquad \text{"}x\text{"} = t \quad \text{"}y(x)\text{"} = P(t)$$

$k>0$: population growth rate is proportional to population
$k<0$: " decay " " " "

Solve this DE!

Chain rule backwards | same as differentials (separable DE.)

(2) Newton's Law of cooling

A = ambient temp.

↑ object with temp $T(t)$

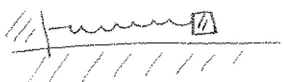
$$\frac{dT}{dt} = k(A - T)$$

"rate of change of temp is proportional to difference between temp and ambient temp"

(3) Spring-mass configuration

m = mass

wall



flat table

$x(t)$ = mass displacement from equilibrium at time t

2^{nd} order DE, from Newton's Law and "linearization"

$$\underbrace{m\,x''(t)}_{\text{mass}\cdot\text{acceleration}} = \text{net forces on mass} = -kx - cx'$$

If you assume force depends only on position x and velocity x', $F = F(x, x')$ and $F(0,0) = 0$ (no net forces at equilibrium) then this is the linear part of the Taylor series for F, i.e. we ignore higher order terms.

k := Hooke's const

c := coef of drag.

<u>Example</u>: Consider the no-drag case,

$$* \qquad x''(t) + 16\,x(t) = 0.$$

Show $x(t) = A\cos 4t + B\sin 4t$ solves this D.E.

Solve the initial value problem for $*$, with initial displacement $= 1$, initial velocity $= 2$

(4) projectiles

y

$$m\,y''(t) = -mg$$

$y(t)$ = height at time t

g = acceleration of gravity $\quad 9.8\ m/s^2$

$$y'' = -g$$ solve it!

$$y' =$$

(5) example.

a) Show $y(x) := \dfrac{1}{c-x}$ solves $\dfrac{dy}{dx} = y^2$

b) solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = y^2 \\ y(1) = 2 \end{cases}$$

(2) Newton's Law of cooling

murder mystery... a body is found at 3 A.M. temp = 85°

by 4 A.M. temp = 80°

65° = A

about when did the murder occur?

$$\frac{dT}{dt} = k(A - T) \quad \ldots$$