Name...... Solutions ....

I.D. number..

Math 2250–4
FINAL EXAM
May 4, 2011

This exam is closed–book and closed–note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. Laplace Transform tables are included with this exam. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** This exam counts for 30% of your course grade. It has been written so that there are 150 points possible, and the point values for each problem are indicated in the right–hand margin. **Good Luck!**

problem	score	possible
1	_____	25
2	_____	15
3	_____	15
4	_____	25
5	_____	25
6	_____	20
7	_____	25
total	_____	150

1) Your father has a "sporty" car of mass 1200 kg and the engine and transmission of this car are able to provide a maximum force of 18,000 N, at all speeds.

1a) Assume the frictional forces on the car from air and road resistance are proportional to velocity, i.e. we are studying a linear drag model. What is the coefficient of friction, and what are its units, if Newton's law implies the car's velocity under the maximum acceleration force above satisfies the differential equation below?

$$v'(t) = 15 - 0.3 \cdot v(t)$$ (5 points)

$$m\frac{dv}{dt} = \text{forces}$$

units: each term is N

$$1200\,v' = 18,000 - cv$$

so [C units is $Ns/m = kg/s$]

$$v' = \frac{18,000}{1200} - \frac{c}{1200}v = 15 - .3v$$

so $\frac{c}{1200} = .3$; $C = (.3)(1200) = 360$

1b) Suppose you're driving your father's car at a raceway, at a velocity of 30 $\frac{m}{s}$, at which point you begin to accelerate at the maximum rate possible, i.e. the car's velocity satisfies the initial value problem

$$v'(t) = 15 - 0.3 \cdot v(t)$$
$$v(0) = 30.$$

We've studied four techniques which apply to find solutions to this initial value problem: first order linear; separable; n^{th}-order linear (Chapter 5); Laplace transform. Use any two of these four techniques above to solve the initial value problem. If you try more than two techniques, indicate clearly which two you wish to have graded. (20 points)

1st order linear:

$$v' + .3v = 15$$
$$(e^{.3t}v)' = 15e^{.3t}$$
$$e^{.3t}v = \int 15e^{.3t}dt$$
$$= \frac{15}{.3}e^{.3t} + C$$
$$e^{.3t}v = 50e^{.3t} + C$$

@$t=0$: $30 = 50 + C$

so $C = -20$

$\div e^{.3t}$: [$v = 50 - 20e^{-.3t}$]

1st order separable.

$$v' = -.3(v - \frac{15}{.3})$$
$$\frac{dv}{dt} = -.3(v - 50)$$
$$\frac{dv}{v - 50} = -.3\,dt$$
$$\int: \ln|v - 50| = -.3t + C_1$$
$$|v - 50| = e^{C_1}e^{-.3t}$$
$$v - 50 = Ce^{-.3t}$$

@$t=0$
$-20 = C$
$\Rightarrow$ [$v = 50 - 20e^{-.3t}$]

n^{th} order linear

$$v' + .3v = 15$$
v_H: $v = e^{rt} \rightarrow r + .3 = 0$; $r = -.3$
$$v_H = C_1e^{-.3t}$$
v_p: try $v_p = C$. $\Rightarrow v' + .3v = .3C = 15 \Rightarrow C = 50$. $\Rightarrow v_p = 50$
$$v = v_p + v_H = 50 + C_1e^{-.3t}$$
$v(0) = 30 \Rightarrow C_1 = -20 \Rightarrow$ [$v = 50 - 20e^{-.3t}$]

Laplace

$$sV(s) - v_0^{\;30} = \frac{15}{s} - .3V(s)$$
$$V(s)(s + .3) = \frac{15}{s} + 30$$
$$V(s) = \frac{15}{s(s + .3)} + \frac{30}{s + .3}$$
$$= \frac{15}{s}\left(\frac{1}{s} - \frac{1}{s + .3}\right)\left(\frac{1}{.3}\right) + \frac{30}{s + .3}$$
$$= 50\left(\frac{1}{s} - \frac{1}{s + .3}\right) + \frac{30}{s + .3}$$
$$V(s) = \frac{50}{s} - \frac{20}{s + .3}$$

[$v(t) = 50 - 20e^{-.3t}$]

2

2a) Use Laplace transform techniques to find the solution to the initial value problem
$$x''(t) + \omega_0^2 \cdot x(t) = F_0 \sin(\omega_0 t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$
(12 points)

$$s^2 X(s) - s x_0 - v_0 + \omega_0^2 X(s) = F_0 \omega_0 \frac{1}{s^2 + \omega_0^2}$$

$$X(s)(s^2 + \omega_0^2) = F_0 \omega_0 \frac{1}{s^2 + \omega_0^2} + s x_0 + v_0$$

$$X(s) = F_0 \omega_0 \frac{1}{(s^2 + \omega_0^2)^2} + x_0 \frac{s}{(s^2 + \omega_0^2)} + \frac{v_0}{\omega_0} \frac{\omega_0}{(s^2 + \omega_0^2)}$$

table:
$$x(t) = F_0 \omega_0 \underbrace{\frac{1}{2\omega_0^3}}(\sin\omega_0 t - \omega_0 t \cos\omega_0 t) + x_0 \cos\omega_0 t + \frac{v_0}{\omega_0} \sin\omega_0 t$$

$$= \frac{F_0}{2\omega_0^2} \sin\omega_0 t - \frac{F_0 t}{2\omega_0} \cos\omega_0 t + x_0 \cos\omega_0 t + \frac{v_0}{\omega_0} \sin\omega_0 t$$

2b) What phenomenon is exhibited by solutions to (2a)? Explain. (3 points)

resonance. no damping & $\omega = \omega_0$ leads to

the sum of a bounded sol'n with

a linearly growing one

3

3) Consider the matrix

$$A := \begin{bmatrix} -2 & 0 \\ 2 & -1 \end{bmatrix}$$

Find a basis of R^2 made out of eigenvectors for this matrix. (15 points)

$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & 0 \\ 2 & -1-\lambda \end{vmatrix} = (2+\lambda)(1+\lambda) \; ; \quad \lambda = -2, -1$$

(or see directly because
matrix is lower $\triangle$ ular).

$\lambda = -2$

$$\begin{array}{cc|c} 0 & 0 & 0 \\ 2 & 1 & 0 \end{array}$$

$$\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

$\lambda = -1$

$$\begin{array}{cc|c} -1 & 0 & 0 \\ 2 & 0 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}$$

$$\vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{(can also see this directly)}.$$

R^2 basis $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}.$

$\lambda = -2 \quad \lambda = -1.$

4) We first encountered a tank cascade in Chapter 1, and this should be the last time we encounter one for at least several months. Consider two tanks. Salt water flows into the first tank at a constant rate of 800 gallons an hour, with a concentration of 1 pound of salt per 200 gallons of water. This first tank maintains a constant volume of 400 gallons by continuously pumping well–mixed water into the second tank, at the same constant rate of 800 gallons/hour. The second tank pumps well–mixed water out at this same rate, maintaining a constant volume of 800 gallons.

$r_i = 800 \frac{g}{h}$ $c_i = \frac{1}{200} \frac{lb}{g}$

$400\ y$ $r = 800\ g/h$

800 $r = 800\ g/h$.
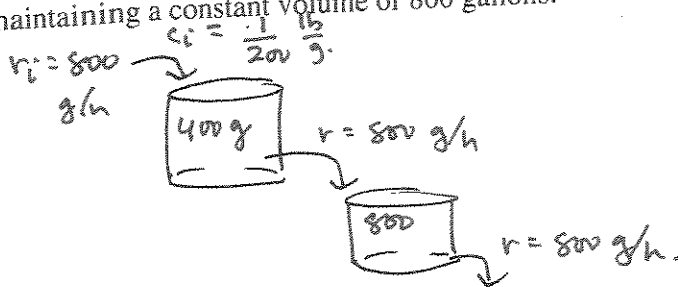
4a) Let $x(t), y(t)$ denote the amount of salt in tanks 1 and 2, respectively. Show that this tank system is modeled by the differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}. \quad = \begin{bmatrix} 4 - 2x \\ 2x - y \end{bmatrix}$$

(5 points)

$$\frac{dx}{dt} = r_i c_i - r_0 c_0 = \frac{800}{200} - 800 \frac{x}{400} = 4 - 2x$$

$$\frac{dy}{dt} = 800 \frac{x}{400} - 800 \frac{y}{800} = 2x - y$$

4b) Use your eigenvalue/eigenvector work from problem (3) to exhibit the general solution to the homogeneous sytem of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(5 points)

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

5

4c) Find a particular solution to the non–homogeneous system of differential equations in (4a). (Hint: find a constant solution.) (5 points)

$$\text{try } x_p = \vec{z} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\vec{0} = \vec{z}' = \begin{bmatrix} -2 & 0 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$+2c_1 = 4 \quad \boxed{c_1 = 2}$$
$$2c_1 - c_2 = 0 \quad \boxed{c_2 = 4}$$

$$\vec{x}_p = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

Could guess this because tanks will flush
& limiting concentration will be input
concentration $\frac{1}{200} \frac{lb}{gall} \Rightarrow$ 2 lb in tank 1
4 lb in tank 2

4d) Suppose there was initially no salt in either tank, so that $x(0) = y(0) = 0$. Solve the resulting initial value problem for (4a). (10 points)

$$\vec{x} = \vec{x}_p + \vec{x}_H$$

$$= \begin{bmatrix} 2 \\ 4 \end{bmatrix} + c_1 e^{-2t}\begin{bmatrix} -1 \\ 2 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$@ t=0 \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -4 \end{bmatrix} = -\begin{bmatrix} -2 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + 2e^{-2t}\begin{bmatrix} -1 \\ 2 \end{bmatrix} - 8e^{-t}\begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

tank 1: $x_1 = 2 - 2e^{-2t}$
tank 2: $x_2 = 4 + 4e^{-2t} - 8e^{-t}$.

5) Consider the following configuration of springs, with positive displacements from equilibrium measured to the right, as indicated.
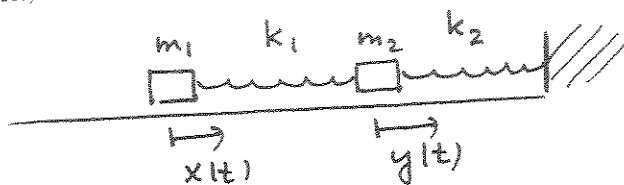
5a) Derive the system of second order differential equations which models this system. Assume there are no frictional or external forces. (5 points)

$$m_1 x'' = k_1(y-x)$$
$$m_2 y'' = -k_1(y-x) - k_2 y$$

5b) Assume that in appropriate units, $m_1 = 2$, $m_2 = 2$, $k_1 = 4$, $k_2 = 6$. Show that in this case the system above reduces to

$$\begin{bmatrix} x''(t) \\ y''(t) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -2x + 2y \\ 2x - 5y \end{bmatrix}$$

(5 points)

$$x'' = -\frac{k_1}{m_1} x + \frac{k_1}{m_1} y$$
$$y'' = \frac{k_1}{m_2} x - \frac{(k_1+k_2)}{m_2} y$$

plug in values

$$x'' = -\frac{4}{2} x + \frac{4}{2} y = -2x + 2y \checkmark$$
$$y'' = \frac{4}{2} x - \frac{10}{2} y = 2x - 5y \checkmark$$

5c) Find the general solution to the system in (5b). Describe the two fundamental modes of vibration. (15 points)

$$\begin{vmatrix} -2-\lambda & 2 \\ 2 & -5-\lambda \end{vmatrix} = (\lambda+5)(\lambda+2) - 4 = \lambda^2 + 7\lambda + 6 = (\lambda+6)(\lambda+1)$$

$\lambda = -6$

$$\begin{array}{cc} 4 & 2 & | & 0 \\ 2 & 1 & | & 0 \end{array}$$

$$\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\omega = \sqrt{-\lambda} = \sqrt{6}$$

$\lambda = -1$

$$\begin{array}{cc} -1 & 2 & | & 0 \\ 2 & -4 & | & 0 \end{array}$$

$$\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\omega = \sqrt{1} = 1.$$

$$\left(c_1 \cos\sqrt{6}t + c_2 \sin\sqrt{6}t\right)\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \left(c_3 \cos t + c_4 \sin t\right)\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_1 \cos(\sqrt{6}t - \alpha) \qquad C_2 \cos(t-\alpha)$$

mass 1 & 2 out of phase, mass 2 has twice the amplitude, $\omega = \sqrt{6}$ rad/sec.

slower, in phase motion, mass 1 has twice the amplitude, $\omega = 1$ rad/sec.

6) Although we usually use a mass–spring configuration to give context for studying second order differential equations, we've also used the rigid–rod pendulum to effectively exhibit several key ideas from this course. Recall that in the undamped version of this configuration, we let the pendulum rod length be L, assume the rod is massless, and that there is a mass m attached at the end on which the vertical graviational force acts with force $m \cdot g$. This mass will swing along circular arcs of signed arclength $s = L \cdot \theta$ from the vertical, where θ is the angle in radians from vertical. The configuration is indicated below. Note that we are considering a pendulum for which the mass (and rod) are able to rotate freely about the fixed end of the rod.
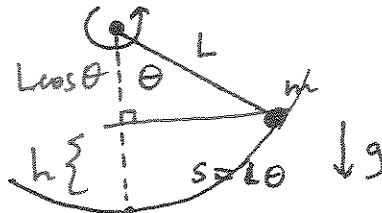
6a) Use the fact that the undamped pendulum system is conservative, i.e. the sum of the kinetic energy from the mass motion and the potential energy from its change in height must be constant once the system is put into motion, to derive the differential equation for $\theta(t)$,

$$\theta''(t) + \frac{g}{L} \cdot \sin\big(\theta(t)\big) = 0.$$

Hint: Since the total energy is constant, its time derivative must be zero.   (10 points)

$$s = L\theta$$

$$\frac{ds}{dt} = L\theta'(t)$$

$$KE = \frac{1}{2}m\left(\frac{ds}{dt}\right)^2 = \frac{1}{2}mL^2\,\theta'(t)^2$$

height above $\theta = 0$ position
is $L - L\cos\theta$

$$PE = mgh = mg(L - L\cos\theta)$$

$$KE + PE = \text{const} = \frac{1}{2}mL^2\,\theta'(t)^2 + mg(L - L\cos\theta)$$

$$\Rightarrow \quad 0 = \frac{d}{dt}(KE + PE) = \frac{1}{2}mL^2\,2\theta'\theta'' - mgL(-\sin\theta)\theta'(t)$$

$$0 = mL\theta'\big[L\theta'' + g\sin\theta\big] = 0 \Rightarrow \boxed{L\theta'' + g\sin\theta(t) = 0}$$

6b) Under the assumption that $|\theta(t)|$ is near zero we linearized the $\sin(\theta)$ term in (6a) to get a linear differential equation. What is this differential equation?   (5 points)

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\sin\theta \approx \theta \quad \text{yields} \quad \theta'' + \frac{g}{L}\theta = 0.$$

6c) Use the linearized differential equation from (6b) to derive the formula we used for the period of a freely oscillating pendulum with small amplitude, in terms of g and L.   (5 points)

$$\omega_0^2 = \frac{g}{L}$$

$$\omega_0 = \sqrt{\frac{g}{L}}$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{L}{g}}}$$

8

7) Assuming damping proportional to velocity, the pendulum equation in (6) is replaced with

(1) $\quad \theta''(t) + c\cdot\theta'(t) + \frac{g}{L}\cdot\sin(\theta(t)) = 0$

with positive constant c.

7a) Explain precisely how this second order damped pendulum differential equation above is related to the first order system of differential equations

If θ solves (1),
let $\quad \theta = x$
$\qquad \theta' = y$

(2) $\quad \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} y \\ -\dfrac{g\sin(x)}{L} - c\cdot y \end{bmatrix}$

(4 points)

then $x' = y$
$y' = \theta'' = -c\theta' - \frac{g}{L}\sin\theta = -cy - \frac{g}{L}x$

so $\begin{bmatrix} x \\ y \end{bmatrix}$ solves (2).

Conversely, if $\begin{bmatrix} x \\ y \end{bmatrix}$ solves (2) then let $\theta(t) = x(t)$, and it will solve (1).

7b) Find all equilibrium solutions of the system (7a), and describe the pendulum configurations they correspond to.

(4 points)

$y = 0$

$-\frac{g}{L}\sin x - cy = 0 \implies \sin x = 0 \implies x = n\pi \qquad n \in \mathbb{Z} \text{ (an integer)}.$

$\qquad\qquad\qquad\uparrow$
$\qquad\qquad\qquad 0$

since $x = \theta$ and $y = \theta'$, these are constant solutions. If θ is odd multiple of π, ~~pendulum~~ mass is at top of circle, if θ is even multiple of π, mass is at bottom.

7c) Consider a special case of the system above, in which the various constants yield the system

$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} y \\ -5\cdot\sin(x) - 2\cdot y \end{bmatrix} \cdot \begin{array}{l} = F(x,y) \\ = G(x,y) \end{array}$

Find the linearized system of differential equations near the the equilibrium solution $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(5 points)

$J = \begin{bmatrix} F_x & F_y \\ G_x & G_y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5\cos x & -2 \end{bmatrix}$

@ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}$

$\boxed{\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}}$

7d) Compute eigenvalues of the linearized system in order to classify the stability and type of equilibrium at the origin. Would you consider this an "underdamped" or "overdamped" pendulum? (6 points)

$$\begin{vmatrix} -\lambda & 1 \\ -5 & -2-\lambda \end{vmatrix} = \lambda(\lambda+2) + 5 = \lambda^2 + 2\lambda + 5$$

$$= (\lambda+1)^2 + 4 = 0$$

$$\lambda = -1 \pm 2i$$

(spiral sink) stable

(solns real & cplx part of $e^{-t}(\cos 2t + i \sin 2t)\vec{v}$
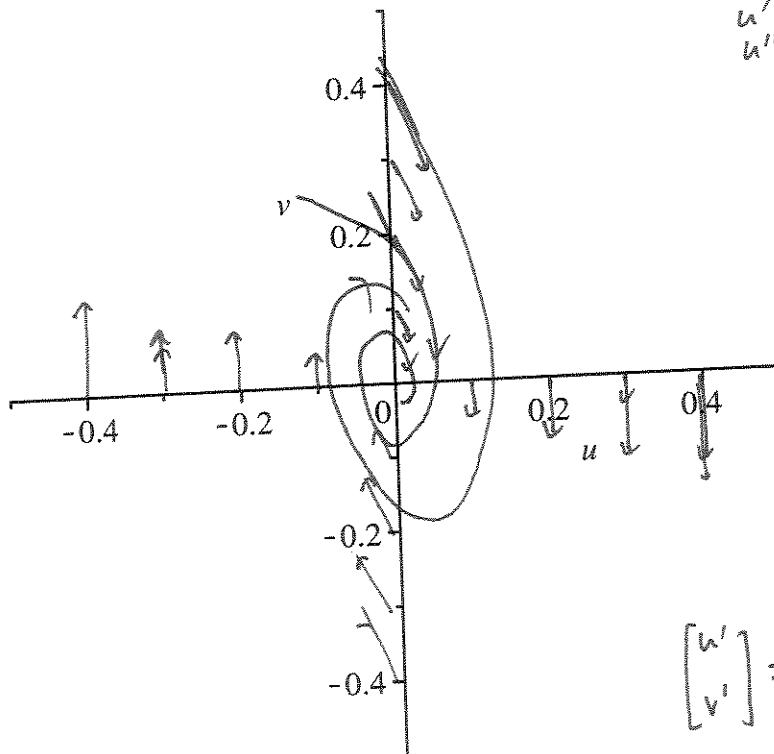
$\lambda = -1+2i$ eigenvectr.

underdamped, because $\Theta(t) = u(t)$ oscillates ∞'ly often.

7e) Use your work in (7d) to sketch a qualitatively correct picture of what the phase portrait for the system in (7c) looks like near the origin. (6 points)

in fact, the

$$u' = v$$
$$u'' = v' = -5u - 2v = -5u - 2u'$$
$$u'' + 2u' + 5u = 0$$
is underdamped.

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

if $\begin{bmatrix} u \\ v \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

tang field $= s \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = s \begin{bmatrix} 0 \\ -5 \end{bmatrix}$

if $\begin{bmatrix} u \\ v \end{bmatrix} = s \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = s \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Table of Laplace Transforms

This table summarizes the general properties of Laplace transforms and the Laplace transforms of particular functions derived in Chapter 7.

Function	Transform	Function	Transform
$f(t)$	$F(s)$	e^{at}	$\dfrac{1}{s-a}$
$af(t)+bg(t)$	$aF(s)+bG(s)$	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
$f'(t)$	$sF(s)-f(0)$	$\cos kt$	$\dfrac{s}{s^2+k^2}$
$f''(t)$	$s^2 F(s)-sf(0)-f'(0)$	$\sin kt$	$\dfrac{k}{s^2+k^2}$
$f^{(n)}(t)$	$s^n F(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$	$\cosh kt$	$\dfrac{s}{s^2-k^2}$
$\displaystyle\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$	$\sinh kt$	$\dfrac{k}{s^2-k^2}$
$e^{at}f(t)$	$F(s-a)$	$e^{at}\cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
$u(t-a)f(t-a)$	$e^{-as}F(s)$	$e^{at}\sin kt$	$\dfrac{k}{(s-a)^2+k^2}$
$\displaystyle\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$	$\dfrac{1}{2k^3}(\sin kt - kt\cos kt)$	$\dfrac{1}{(s^2+k^2)^2}$
$tf(t)$	$-F'(s)$	$\dfrac{1}{2k}\sin kt$	$\dfrac{s}{(s^2+k^2)^2}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$\dfrac{1}{2k}(\sin kt + kt\cos kt)$	$\dfrac{s^2}{(s^2+k^2)^2}$
$\dfrac{f(t)}{t}$	$\displaystyle\int_s^\infty F(\sigma)\,d\sigma$	$u(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t),\quad$ period p	$\dfrac{1}{1-e^{-ps}}\displaystyle\int_0^p e^{-st}f(t)\,dt$	$\delta(t-a)$	e^{-as}
1	$\dfrac{1}{s}$	$(-1)^{\llbracket t/a\rrbracket}$ (square wave)	$\dfrac{1}{s}\tanh\dfrac{as}{2}$
t	$\dfrac{1}{s^2}$	$\left\llbracket\dfrac{t}{a}\right\rrbracket$ (staircase)	$\dfrac{e^{-as}}{s(1-e^{-as})}$
t^n	$\dfrac{n!}{s^{n+1}}$		
$\dfrac{1}{\sqrt{\pi t}}$	$\dfrac{1}{\sqrt{s}}$		
t^a	$\dfrac{\Gamma(a+1)}{s^{a+1}}$		