Math 2250-4   §3.1-3.2   1

Friday 2/4   Linear systems

HW for Fri Feb. 11

3.1   1, 4, 11, 16, 17, 19, 23, 29, 32, 33, 34

3.2   7, 8, 9, 13, 17, 29, 30

3.3   13, 20, 33, 39

3.4   3, 5, 7, 10, 13, 19, 31, 32, 34, 39, 40, 44

do the problems above by hand, but I encourage software checks (e.g. Maple, Matlab, etc) hand in techology checks for

3.3   20

3.4   13

• Do examples on pages 3, 5 Wed. notes. Keep in mind the general set-up below:

Linear system of m equations in n unknowns:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{array}\right\} LS$$

<u>Goal</u>: find all (unknown) solution vectors $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ (for which all m eqtns hold simultaneously)

the collection of these solutions is called the <u>solution</u> <u>set</u>

The rectangle of coefficients

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is called the <u>coefficient</u> <u>matrix</u> A

If we adjoin the right-hand side vector:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & b_m \end{array}\right]$$

this is the <u>augmented</u> <u>matrix</u> $A|\vec{b}$

We may do the following equation operations without changing the solution set:

1. Interchange 2 eqtns
2. multiply an eqtn by a non-zero constant
3. replace an eqtn by its sum with any multiple of another eqtn

[we can combine several of these into a single step, e.g. replace an eqtn with the sum of a non-zero mult of it, and a multiple of another eqtn]

So when we work synthetically with the augmented matrix we may do the corresponding 3 <u>elementary row operations</u> without changing sol'n set:

1. interchange 2 rows ("swap")
2. multiply a row by a non-zero constant
3. replace a row by its sum with a multiple of another row.

Example 5 p. 164, illustrating Gaussian elimination for a big system)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 &= 10 \\ 2x_1 - 4x_2 + 8x_3 + 3x_4 + 10x_5 &= 7 \\ 3x_1 - 6x_2 + 10x_3 + 6x_4 + 5x_5 &= 27 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 2 & -4 & 8 & 3 & 10 & 7 \\ 3 & -6 & 10 & 6 & 5 & 27 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1+R_2 \\ -3R_1+R_3 \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 2 & -1 & 8 & -13 \\ 0 & 0 & 1 & 0 & 2 & -3 \end{array}\right]$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 2 & -1 & 8 & -13 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -2R_2+R_3 \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & -1 & 4 & -7 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -R_3 \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array}\right]$$

at this point we can backsolve to find sol'n:

$x_5 = t$
$x_4 = 7 + 4t$
$x_3 = -3 - 2t$
$x_2 = s$
$x_1 = 10 - t - 2(7+4t) - 3(-3-2t) + 2s$
$\quad = 5 + t(-3) + s(2)$

So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 7 \\ 0 \end{bmatrix} + s\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ -2 \\ 4 \\ 1 \end{bmatrix}$, $s, t \in \mathbb{R}$

row echelon form

1. all "zero rows" (having all 0's) lie beneath all "non-zero rows"
2. the leading non-zero entry in each non-zero row lies strictly to the right of the leading non-zero entry in the previous row.

or proceed to reduced row-echelon form, by cleaning from the bottom right, moving up & to the left

$$\begin{array}{r} 2R_3+R_1 \\ \\ \\ \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 3 & 0 & 9 & -4 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array}\right]$$

$$\begin{array}{r} 3R_2+R_1 \\ \\ \\ \end{array} \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array}\right]$$

reduced row echelon form 1, 2 and

3. each leading non-zero entry is a 1 (called a "leading 1")
4. Every column with a leading 1 has zeroes everywhere else, i.e. above as well as below

now, backsolving is easier!

$x_5 = t$
$x_4 = 7 + 4t$
$x_3 = -3 - 2t$
$x_2 = s$
$x_1 = 5 - 3t + 2s$

same ans!

> With Gaussian elimination we can reduce any matrix to its r.r.e.f.