Name...

Student I.D. number..

Math 2250-4
Exam 2
March 31, 2011

The exam is closed book and closed note, but a scientific calculator is allowed and may be useful. Calculators which can do linear algebra or solve differential equations are not allowed. In order to receive partial credit or full credit all work must be shown.   There are 100 points possible on the test, and the point values of each problem are indicated in the right margin. You may wish to ration your time accordingly. GOOD LUCK!

Score: **Possible**

1_____   30

2_____   40

3_____   20

4_____   10

Total_____   100

1) Here is a matrix A (on the left), and on the right is its reduced row echelon form:

$$A := \begin{bmatrix} 1 & -2 & 1 & -1 & -1 \\ -2 & 4 & -2 & 2 & 2 \\ 1 & -2 & 0 & 2 & 1 \\ 3 & -6 & 2 & 2 & 1 \end{bmatrix} \qquad reduced := \begin{bmatrix} 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1a) Find all solutions to the homogeneous linear equation Ax=0.

(8 points)

1b) Exhibit a basis for the solution space you found in part 1a).

(4 points)

1c) What is the dimension of this solution space?

(3 points)

Here again are the matrix A and its reduced row echelon form:

$$A := \begin{bmatrix} 1 & -2 & 1 & -1 & -1 \\ -2 & 4 & -2 & 2 & 2 \\ 1 & -2 & 0 & 2 & 1 \\ 3 & -6 & 2 & 2 & 1 \end{bmatrix} \qquad reduced := \begin{bmatrix} 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1c) Express the fifth column of A as a linear combination of the preceding columns.

(5 points)

1d) Find a basis for the column space of A (the span of the columns of A). Justify why your basis satisfies the two necessary conditions for it to be a basis of the column space.

(10 points)

2) Consider the differential equation below, which could model a mechanical oscillation or electrical circuit problem.

$$x''(t) + 4 \cdot x'(t) + 13 \cdot x(t) = 0.$$

2a) If this differential equation was modeling an unforced damped mass-spring configuration such as we've studied, and if the mass was 2 kg, determine the damping coefficient and (Hooke's) spring constant. Include correct mks units.

(6 points)

2b) Find a basis for the solution space to this homogeneous linear differential equation.

(10 points)

2c) What sort of damping is exhibited by solutions to this differential equation?

(4 points)

2d) Use the solutions you found in (2b) and Chapter 5 techniques to solve the general initial value problem

$$x''(t) + 4 \cdot x'(t) + 13 \cdot x(t) = 0.$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

(10 points)

2e) Re-solve the initial value problem in (2d), using Laplace transforms.

(10 points)

3) Consider the initial value problem below, which could arise as a forced mass-spring (or electrical circuit) problem.

$$x''(t) + 9 \cdot x(t) = 5 \cdot \cos(2 \cdot t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

3a) Use Laplace transform techniques to solve this problem.

(15 points)

3b) Do solutions to this differential equation exhibit resonance? Explain.

(5 points)

4) Consider the sinusoidal function
$$x(t) = 3 \cdot \cos(0.2 \cdot t) - \sin(0.2 \cdot t).$$

4a) Assuming t measures time in seconds, compute the angular frequency, frequency, and period for this function. Use correct units.

(5 points)

4b) Express $x(t)$ in amplitude-phase form, $x(t) = C \cdot \cos(.2 \cdot t - \alpha)$. You may specify α in terms of an inverse trig function, if you don't have a calculator to compute a decimal value. Also specify the time delay δ of this function, relative to $C \cdot \cos(.2 \cdot t)$. Use correct units.

(5 points)