MATHEMATICS 2250-4
Differential Equations and Linear Algebra
SYLLABUS
Spring semester 2011

when: MTWF 10:45-11:35
where: LCB 219

instructor: Prof. Nick Korevaar
office: LCB 204
telephone: 581-7318
email: korevaar@math.utah.edu

office hours: MWF 2:30-3:20, and by appointment

sections:
instructor: Victor Camacho
home page: www.math.utah.edu.edu/ camacho
2250-5: Th 10:45-11:35, LCB 219
2250-6: Th 11:50-12:40, JTB 140

course home page: www.math.utah.edu/ ~korevaar/spring11

text: *Differential Equations and Linear Algebra*
by C.Henry Edwards and David E. Penney
ISBN = 9780558141066 (custom edition)

prerequisites: Math 1210-1220 (or 1250-1260 or 1270-1280, i.e. single-variable calculus.) You are also expected to have learned about vectors and parametric curves (Math 2210 or Physics 2210 or 3210). Practically speaking, you are better prepared for this course if you've had multivariable calculus (1260, 1280, 2210, or equivalent), and if your grades in the prerequisite courses were above the "C" level.

course outline: This course is an introduction to differential equations, and how they are used to model problems arising in engineering and science. Linear algebra is introduced as a tool for analyzing systems of differential equations, as well as standard linear equations. Computer projects will be assigned to enhance the material.

We will cover most of chapters 1-10 in the Edwards-Penney text, in addition to two supplementary sections which cover RLC electrical circuits and additional Laplace transform material.

The course begins with first order differential equations, a subject which you touched on in Calculus. Recall that a differential equation is an equation involving an unknown function and its derivatives, that such equations often arise in science, and that the order of a differential equation is defined to be the highest order derivative occuring in the equation. The goal is to understand the solution functions to differential equations since these functions will be describing the scientific phenomena which led to the differential equation in the first place. In chapters one and two of Edwards-Penney we learn analytic techniques for solving certain first order DE's, the geometric meaning of graphs of solutions, and the numerical techniques for approximating solutions which are motivated by this geometric interpretation. We will carefully study the logistic population growth model from mathematical biology and various velocity-acceleration models from physics.

At this point in the course we will take a four week digression from differential equations, to learn the fundamentals of linear algebra in chapters 3-4. You need a basic understanding of this field of mathematics in order to talk meaningfully about the theory of higher order linear DE's and of systems of linear DE's. Chapter 3 starts out with matrix equations and the Gauss-Jordan method of solution. When you see such equations in high-school algebra you might be thinking of intersecting lines in the plane, or intersecting planes in space, or ways to balance chemical reactions, but the need to understand generally how to solve such equations is pervasive in science. From this concrete beginning we study abstract vector spaces in chapter 4. This abstract theory is useful not only to the understanding of Euclidean space $\mathbf{R}^n$, but it is also the framework which allows us to understand solution spaces to systems of linear differential equations.

In Chapter 5 we will study the theory of second order and higher-order linear DE's, and focus on the ones which describe basic mechanical vibrations and electrical circuits. You may have been introduced to these equations in Calculus, and we will treat them more completely now, including a careful study of forced oscillations and resonance.

After Chapter 5 we will jump ahead to Chapter 10, the Laplace transform. This "magic" transform converts linear differential into algebraic equations, which you can solve before "inverse-transforming". You will have to see it in action to appreciate it. You will see, for example, that this method gives a powerful way to study forced oscillations in the physically important cases that the forcing terms are square waves or other discontinuous functions. Electrical engineers often use Laplace transform techniques to understand circuits.

To succesfully model more complicated physical systems like shaking in multi-story buildings, one needs the framework of linear systems of differential equations. The linear algebra of eigenvalues and eigenvectors is developed in chapter 6 as a tool for studying systems of DE's in chapter 7, where for example eigenvalues and eigenvectors will be related to fundamental modes for mass-spring systems. We will also understand these systems of DEs from the point of view of Laplace transforms.

Chapter 8, about matrix exponentials, shows the matrix algebra leads to beautiful formulas for solutions to constant coefficient linear systems of differential equations. We will only cover this chapter if time permits.

Our final course topic is Chapter 9, nonlinear systems and phenomena. This topic ties together many different ideas, as we discuss the stability of equilibria, based on linearization, and consider the geometry of solution trajectories in the phase portrait of autonomous systems. Many of the most interesting (complicated) dynamical systems on earth are non-linear; perhaps the most famous current example of this is global climate change. (We won't be able to study that topic in this course.)

sections: The purpose of the sections is to work examples and problems related to the course and exam material. In addition, the two course midterms will be administered during your sections.

homework: when assigned from the text, homework will be collected each week on Fridays, and a large proportion of the problems will be graded. You will know the assignment due on Friday by the previous Friday. You are encouraged to make friends and study groups for discussing homework, although you will each hand in your own papers (and copying someone elses work won't be productive).

The Thursday problem sessions mentioned at the beginning of this syllabus should be helpful for many of you. Note also that the Math Department Lab is located down the hall from that room, in the Rushing Student Center, between JWB and LCB. Initial tutorial times will be announced.

More information about the lab can be found at the Math Department web page for undergraduates, www.math.utah.edu/ugrad .

The Math tutoring center is in the Rushing Student Center, in the basement between LCB and JWB on President's Circle. You will be able to find tutors there who can help with Math 2250 homework (8 a.m.- 8 p.m. Monday-Thursday and 8 a.m.- 4 p.m. on Fridays). The page *www.math.utah.edu/ugrad/mathcenter.html* has more information.

computer projects: There will be several computer projects assigned during the semester, related to the classroom material. They will be written in the software package MAPLE. In addition, you will be asked to use this computer software to check various homework calculations from throughout the course.

There is a Math Department Computer Lab at which you all automatically have accounts, and there are other labs around campus where MAPLE is also available, for example at the College of Engineering and Marriott Library.

There will tutoring center support for these projects (and for your other homework) as well. There will be introductory sessions to the Math lab and to Maple. These will be held in LCB 115, and times will be announced during the first week of classes.

exams: There will be two in-class midterms (closed book, scientific calculator only), as well as a final exam. The dates are as follows:

exam 1: Thursday February 17. Probable course material is chapters 1-3.
exam 2: Thursday March 31 . Probable course material is chapters 4-5,10.
Final Exam: 10:30-12:30 Wednesday May 4 in our classroom LCB 219. The exam will cover the entire course, although the material after the second midterm (Chapters 6-7,9) can be expected to have extra weight. This is the University-scheduled time.

grading: Each midterm will count for 20% of your grade, the book homework and the projects will count for a total of 30%, and the final exam will make up the remaining 30% of your grade. The value of carefully working the homework problems and projects is that mathematics (like anything) must be practiced and experienced to really be learned. **Note:** In order to receive a grade of at least "C" in the course you must earn a grade of at least "C" on the final exam.

University dates to keep in mind: Monday January 24 is the last day to add this class, Wednesday January 19 is the last day to drop it. Friday March 5 is the last day to withdraw. (All of these dates are easy to find from the University home page.)

ADA statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed,
daily subject matter approximated

M	10 Jan	1.1	differential equations and mathematical models
T	11 Jan	1.2	integral as general and particular solutions
W	12 Jan	1.3	slope fields and solution curves
F	14 Jan	1.4	separable differential equations
M	17 Jan	none	Martin Luther King Day
T	18 Jan	1.4	continued
W	19 Jan	1.5	linear first order differential equations
F	21 Jan	1.5	continued
M	24 Jan	2.1	population models
T	25 Jan	2.1-2.2	equilibrium solutions and stability
W	26 Jan	2.2-2.3	and acceleration-velocity models
F	28 Jan	2.3	continued
M	31 Jan	2.4-2.6	numerical solution approximations
T	1 Feb	3.1-3.2	introduction to linear systems
W	2 Feb	3.1-3.3	matrices and gaussian elimination
F	4 Feb	3.3	reduced row echelon form
M	7 Feb	3.4	matrix operations
T	8 Feb	3.4-3.5	matrix inverses
W	9 Feb	3.5	continued
F	11 Feb	3.6	determinants
M	14 Feb	3.6	continued
T	15 Feb	4.1-4.2	3-space as a vector space
W	16 Feb	4.2-4.3	n-space, subspaces, linear combinations
Th	17 Feb	1-3.6	EXAM 1
F	18 Feb	4.3-4.4	and dependence and independence
M	21 Feb	none	President's Day
T	22 Feb	4.4	and bases and dimension
W	23 Feb	4.5	row and column space
F	25 Feb	4.7	general vector spaces
M	28 Feb	5.1	intro to second order linear differential equations
T	1 Mar	5.2-5.3	general solutions to linear DEs
W	2 Mar	5.3	homogeneous equations with constant coefficients
F	4 Mar	5.3-5.4	complex roots and mechanical vibrations

M	7 Mar	5.4	continued
T	8 Mar	5.5	nonhomogeneous equations and undetermined coefficients
W	9 Mar	5.6	forced oscillations and resonance
F	11 Mar	5.6	continued
M	14 Mar	EP 3.7	RLC circuits
T	15 Mar	10.1-10.2	Laplace transforms and initial value problems
W	16 Mar	10.2-10.3	partial fractions and translations
F	18 Mar	10.4-10.5	derivatives, integrals and products
M	21 Mar	none	Spring break
T	22 Mar	none	Spring break
W	23 Mar	none	Spring break
F	25 Mar	none	Spring break
M	28 Mar	10.4-10.5	periodic and piecewise continuous input functions
T	29 Mar	EP 7.6	impulses and delta functions
W	30 Mar	4.5-EP7.6	review
Th	31 Mar	4.5-EP 7.6	EXAM 2
F	1 Apr	6.1	eigenvalues and eigenvectors
M	4 Apr	6.1-6.2	diagonalization of matrices
T	5 Apr	6.2-6.3	discrete dynamical systems
W	6 Apr	6.3	applications
F	8 Apr	7.1	existence and uniqueness for first order systems of DEs
M	11 Apr	7.2-7.3	eigenvalue-eigenvector method for linear systems of DEs
T	12 Apr	7.3	applications of 1st order ODE systems
W	13 Apr	7.4	spring systems
F	15 Apr	7.4	forced undamped spring systems - practical resonance
M	18 Apr	7.5	chains for defective eigenspaces
T	19 Apr	9.1-9.2	equilibria, stability and the phase plane
W	20 Apr	9.2	linearization near equilibrium solutions
F	22 Apr	9.2-9.3	ecological models
M	25 Apr	9.3	and competition models
T	26 Apr	9.4	nonlinear mechanical systems
W	27 Apr	review	review
W	4 May	FINAL EXAM	entire course, 10:30-12:30 a.m., in our classroom