Math 2250
Earthquake project
April 2001

Enter your name here: ?????????????????????????????

Project 3. Please fill in all areas below marked ????? and solve problems 3.1 to 3.6. The problem headers:

_______ PROBLEM 3.1. BUILDING MODEL FOR AN EARTHQUAKE.
_______ PROBLEM 3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_______ PROBLEM 3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
_______ PROBLEM 3.4. PRACTICAL RESONANCE.
_______ PROBLEM 3.5. EARTHQUAKE DAMAGE.
_______ PROBLEM 3.6. THREE FLOORS.

```
> with(DEtools):with(plots):with(linalg):
```

3.1. BUILDING MODEL FOR AN EARTHQUAKE.
Refer to the textbook of Edwards-Penney, section 7.4, page 437. Consider a building with 7 floors.

Let the mass in slugs of each story be m=1000.0 and let the spring constant be k=10000.0 (lbs/foot). Define the 7 by 7 mass matrix M and Hooke's matrix K for this system and convert Mx''=Kx into the system x''=Ax where A is defined by textbook equation (1) , page 437.

PROBLEM 3.1
Find the eigenvalues of the matrix A to six digits, using the Maple command "eigenvals(A)." Justify in particular that all seven eigenvalues are negative by direct computation.

```
# Sample Maple code for a model with 4 floors.
# Use maple help to learn about augment, stackmatrix, diag, transpose, evalm.
 A1:=diag(-20,-20,-20,-10):
 A2:=augment(vector([0,0,0,0]),stackmatrix(diag(1,1,1),matrix([[0,0,0]]))):
 A3:=transpose(A2):
 A:=evalm(A1+10*A2+10*A3);
 evalf(eigenvals(A));
```

```
> # Problem 3.1
```

3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17, page 437.

PROBLEM 3.2.
Find the natural angular frequencies omega=sqrt(-lambda) for the seven story building and also the corresponding periods
2PI/omega, accurate to six digits. Display the answers in a table .

```
# Sample code for a 4x3 table.
```

```
# Use maple help to learn about printf.
 ev:=[-38.3,-33.4,-26.2,-17.9]:  n:=4:
 Omega:=lambda -> sqrt(-lambda):
 format:="%10.6f  %10.6f  %10.6f\n":
 seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n);
```

```
> # Problem 3.2
```

3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
Consider the forced equation x'=Ax+cos(wt)b where b is a constant vector. The earthquake's ground vibration is accounted for by the extra term cos(wt)b, which has period T=2Pi/w.
The solution x(t) is the 7-vector of excursions from equilibrium of the corresponding 7 floors. Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form x(t)=cos(wt)c for some vector c that depends only on A and b.

PROBLEM 3.3.
Define b:=(1/4)*w*w*vector([1,1,1,1,1,1,1]): in Maple and find the vector c in the undetermined coefficients solution x(t)=cos(wt)c. Vector c depends on w. As outlined in the textbook, vector c can be found by solving the linear algebra problem -w^2 c = Ac + b; see page 433. Don't print c, as it is too complex; instead, print c[1] as an illustration.

```
# Sample code for defining  b and A, then solving for  c  in the  4-floor case.
  w:='w':
  b:=0.25*w*w*vector([1,1,1,1]):
  A1:=diag(-20,-20,-20,-10):
  A2:=augment(vector([0,0,0,0]),stackmatrix(diag(1,1,1),matrix([[0,0,0]]))):
  A:=evalm(A1+10*A2+10*transpose(A2));
  c:=linsolve(evalm(A+w*w*diag(1,1,1,1)),-b):
  evalf(c[1]);
```

```
> # PROBLEM 3.3
```

3.4 PRACTICAL RESONANCE.
Consider the forced equation x'=Ax+cos(wt)b of 3.3 above with b:=0.25*w*w*vector([1,1,1,1,1,1,1]).
Practical resonance can occur if a component of x(t) has large amplitude compared to the vector norm of b. For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

PROBLEM 3.4.
Let Max(c) denote the maximum modulus of the components of vector c. Plot g(T)=Max(c(w)) with
w=T/(2*Pi)
for periods T=0 to T=6, ordinates Max=0 to Max=10, the vector c(w) being the answer produced in 3.3 above.
Compare your figure to the textbook Figure 7.4.18, page 438.

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about "norm."
  with(linalg):
```

```
Max:= c -> norm(c,infinity);
B:=w*w*diag(1,1,1,1): b:=0.25*w*w*vector([1,1,1,1]):
C:=ww -> subs(w=ww,linsolve(evalm(A+B),-b)):
plot(Max(C(2*Pi/r)),r=0..6,0..10);
```

```
> # PROBLEM 3.4
```

3.5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of 3.4 can be used to detect the likelihood of earthquake damage for a given
ground vibration of period T. A ground vibration (1/4)cos(wt), T=2*Pi/w, will be assumed, as in 3.4.

PROBLEM 3.5.
Replot the amplitudes in 3.4 for periods 0 to 6 and amplitudes 5 to 10. Determine approximate ranges for the period T such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

```
> # PROBLEM 3.5
> # Plot it.
> # Print period ranges ???.
```

3.6. THREE FLOORS.
Consider a building with only three floors each weighing 20 tons. Assume each floor corresponds to a restoring
Hooke's force with constant k=4 tons/foot. Assume that ground vibrations from the earthquake are modeled by
(1/4)cos(wt) with period T=2*Pi/w (same as the 7-floor model above).

PROBLEM 3.6.
Model the 3-floor problem in Maple. Plot the maximum amplitudes against the period 0 to 6 and amplitude
4 to 10. Determine from the graphic the period ranges which cause the amplitude plot to spike above 4 feet.

```
> # PROBLEM 3.6
> # Define k=??? and m=???, then matrix A=???.
> # Amplitude plot for T=0..6,C=4..10
> # From the graphics, T-ranges are ????
  # which give amplitude spikes above 4 feet. These are
  # determined by mouse-clicks on the graph, so they
  # are approximate values only.
>
```