Exercise 4) The text homework problems 2.4.4, 2.5.4, 2.6.4 are canceled. However, you will work on a similar set of "by hand" problems in the lab. In preparation for that, use the Euler, Improved Euler, and Runge Kutta pseudo-code below to estimate e^1, as the solution to the initial value problem that we've used exclusively in these notes, namely

$$y'(x) = y$$
$$y(0) = 1.$$

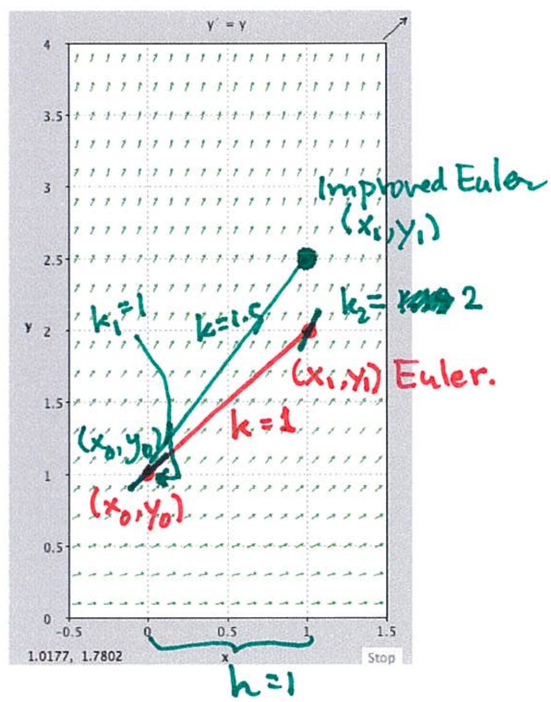
(In the lab you will be studying different initial value problems). In this exercise use just one step, $n = 1$, of size $h = 1$. Also, use the slope fields to illustrate how you find and use the various $k - values$ which are rates of changes that get represented as slopes in the solution graphs. Take one step to get from the initial point $(x_0, y_0) = (0, 1)$ to the final point $(x_1, y_1) = (1, ??)$.

4a) Euler pseudocode for

$$y'(x) = f(x, y) = y$$
$$y(x_0) = y_0$$

$k = f(x_j, y_j)$
$x_{j+1} = x_j + h$
$y_{j+1} = y_j + h\,k.$
(answer is $(x_1, y_1) = (1, 2).$)

Euler: for h=1
$x_0 = 0$
$y_0 = 1$
$k = f(0,1) = 1$
$x_1 = 1$
$y_1 = y_0 + hk$
$= 1 + 1\cdot 1 = 2$
$(x_1, y_1) = (1, 2)$



4b) Improved Euler pseudocode:
$k_1 = f(x_j, y_j)$
$k_2 = f(x_j + h, y_j + h\,k_1)$
$k = \frac{1}{2}(k_1 + k_2)$
$x_{j+1} = x_j + h$
$y_{j+1} = y_j + h\,k$
(answer is $y_1 = 2.5$)

$(x_0, y_0) = (0, 1)$
$k_1 = f(0,1) = 1$
$k_2 = f(1, 1+1) = f(1,2) = 2$
$k = \frac{1}{2}(1+2) = 1.5$
$x_1 = 1$
$y_1 = y_0 + hk$
$= 1 + 1\cdot(1.5)$
$= 2.5$
$(x_1, y_1) = (1, 2.5)$

4c) Runge-Kutta pseudocode:

$k_1 = f(x_j, y_j)$ ← left endpoint slope

$k_2 = f\left(x_j + \frac{h}{2}, y_j + \frac{h}{2} k_1\right)$

$k_3 = f\left(x_j + \frac{h}{2}, y_j + \frac{h}{2} k_2\right)$ } mid points slopes

$k_4 = f(x_j + h, y_j + h\, k_3)$

$k = \frac{1}{6}(k_1 + 2\,k_2 + 2\,k_3 + k_4)$ ← right endpoint slope

$x_{j+1} = x_j + h$

$y_{j+1} = y_j + h\,k$

(answer is $y_1 = 2.708$)

$(x_0, y_0) = (0, 1)$

$k_1 = f(x_0, y_0) = 1$

$k_2 = f(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} k_1)$

$\quad = f(.5, 1 + .5) = f(.5, 1.5) = 1.5$

$k_3 = f(0 + .5, 1 + .5(1.5))$

$\quad = f(.5, 1.75) = 1.75$

$k_4 = f(1, 1 + 1 \cdot 1.75)$

$\quad = f(1, 2.75) = 2.75$

$k = \frac{1}{6}(1 + 2(1.5) + 2(1.75) + 2.75)$

$\quad = 1.708$

$x_1 = 1$

$y_1 = y_0 + hk$

$\quad = 1 + 1.708$

$\quad = 2.708\ldots$

$(x_1, y_1) = (1, 2.708\ldots)$

e actually equals 2.718...