Math 2250-004
Week 15 concepts and homework
section 7.4
Due Friday December 9 at noon

7.3) input-output modeling
7.3: 3, 13, 29, 31, **34**, 36. In **34** you may use technology to find the eigendata. (This problem was postponed from last week.)

7.4) Second order systems of differential equations arising from conservative systems. Identifying fundamental modes and natural angular frequencies; forced oscillation problems and the potential for practical resonance when the forcing frequency is close to a natural frequency.
7.4: **2**, 3, **8**, **12**, 13, **14**, **16**, **18** .

w14.1) This is a continuation of **2**, **8**. Now let's force the spring system in problem 2, with a sinusoidal force on the first mass at (variable) angular frequency ω, as in the slightly different text example on pages 440-442. Thus we consider the system

$$\begin{bmatrix} x''(t) \\ y''(t) \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + F_0 \cos(\omega t) \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

a) Find a particular solution of the form

$$\underline{\boldsymbol{x}}_P(t) = \cos(\omega t)\underline{\boldsymbol{c}}.$$

Hint: Plug this guess into the differential equation. You will notice that each term simplifies to some vector times the function $\cos(\omega t)$. Thus, after you factor out the $\cos(\omega t)$ term you are left with a matrix equation to solve for $\underline{\boldsymbol{c}} = \underline{\boldsymbol{c}}(\omega)$. You will get formulas analogous to equations (34, 35) in section 7.5, except your c_1, c_2 will blow up at $\omega = 1, 3$, the natural frequencies for this problem.

b) The general solution to this forced oscillation problem is the particular solution from part (a), plus the general solution to the homogeneous problem, which you found in problem (2). In a physical problem with a slight amount of damping but the same masses and spring constants, the particular solution would be close to the one you found in part (a), and the homogeneous solutions would be close to the ones you found in problem 2, except that they would be (slowly) exponentially decaying because of the damping. Thus the particular solution would be the steady periodic solution, and the homogeneous solution would be transient. By plotting the magnitude $\|\underline{\boldsymbol{c}}(\omega)\| = \sqrt{c_1(\omega)^2 + c_2(\omega)^2}$ as a function of ω, you create a "practical resonance" chart analogous to those we created in Chapter 5. Create such a plot, for the angular frequency range $0 \le \omega \le 5$. Use $F_0 = 4$. Your plot should look like Figure 7.4.10, except your magnitude function will peak at $\omega = 1, 3$.

w14.2) This is a continuation of **18**. In physics you learn that you can recover the final velocities from the initial ones in a conservative problem like 18 by equating the initial momentum $m_1 v_0$ to the final momentum $m_1 v_1 + m_2 v_2$ and the initial kinetic energy $\frac{1}{2} m_1 v_0^2$ to the final kinetic energy $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$, and solving this system of equations for v_1 and v_2. Carry this procedure out for the data in **18** and show that your answer agrees with your work in that problem.