Math 2250-004

Week 13 concepts and homework, due Wednesday November 30 at 6:00 p.m.

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

10.5: *Piecewise continuous input functions:*
10.5: 3, 7, 11, 13, **31**, 32

10.4: using convolution integrals to find inverse Laplace of F(s)G(s); using the differentiation theorem to find the Laplace transform of $t \cdot f(t)$*; using these (and other) techniques to solve linear differential equations.*

w13.1a) Compute the convolution $f*g(t)$ for $f(t) = t$ and $g(t) = t^2$ and use the Laplace transform table to show that $\mathcal{L}\{f*g(t)\}(s) = F(s)G(s)$ in this case. Be clever about which order you write the *f*, *g* terms in the convolution integral, as one way is easier than the other.
b) Repeat part a for the example $f(t) = t, g(t) = \sin(k\,t)$.

10.4: 2, 3, 9, **36**, **37**, 38. Note: These two problems are very quick applications of the convolution table entry for Laplace transforms.

w13.2) Redo 10.5.31 using the convolution table entry to find the solution $x(t)$ to the initial value problem:

$$x''(t) + 4\,x(t) = f(t)$$
$$x(0) = 0$$
$$x'(0) = 0$$

where the forcing function is given by

$$f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases} .$$

EP 7.6 Engineering applications: Duhamel's Principle and delta function forcing.
EP 7.6 1, **2**

Chapter 6.1-6.2:

6.1: finding eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional).
6.1: 7, 17, 19, 25

6.2: diagonalizability: recognizing matrices for which there is an eigenbasis, and those for which there is not one. Writing the eigenbasis equation as AP = PD where P is the matrix of eigenvector columns, and D is the diagonal matrix of eigenvalues.
6.2: 3, 9, 21, 23.

w13.3) Find eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional), for the following matrices. You can check your work with technology, but you don't have to hand in the technology check.

a) $A := \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$

b) $B := \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$

c) $C := \begin{bmatrix} 2 & 9 & 3 \\ -2 & -5 & 0 \\ 2 & 6 & 1 \end{bmatrix}$

d) $E := \begin{bmatrix} 1 & 6 & 6 \\ 0 & -1 & -2 \\ 0 & 4 & 5 \end{bmatrix}$

e) $F := \begin{bmatrix} 5 & 3 & -9 \\ -4 & -5 & 4 \\ 4 & 2 & -7 \end{bmatrix}$. In this problem you may use technology to compute and factor the characteristic polynomial, and to find the eigenspace bases. Be careful with what the technology is telling you.

w13.4)
a) Which of the matrices in 13.3 are diagonalizable and which are not?
b) For the diagonalizable matrices *A*, *C* above, verify the diagonalizing identity $P^{-1} A P = D$ by checking the equivalent and easier to check equation $A P = P D$.
c) Explain what happens to the diagonal matrix D of eigenvalues for a diagonalizable matrix, if you change the order of the eigenvector columns in *P*, in the identity $A P = P D$. Explain.