Name + UID: Solutions **Section (circle one): C.Wei S.Bagley**

Math 2250-4
Super Quiz 2
November 1, 2013

1a) What is the <u>span</u> of a collection of vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$?

all $\vec{w}$ s.t. $\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$, $c_1, c_2 \ldots c_n \in \mathbb{R}$ (1 point)

i.e. the collection of all linear combinations

1b) What does it mean for vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ to be <u>linearly independent</u>?

$c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n = \vec{0} \Rightarrow c_1 = c_2 = \cdots = c_n = 0$ (1 point)

1c) What is a <u>basis</u> for a vector space/subspace W ?

a collection of vectors $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n$ that spans W, and that is ~~are~~ linearly independent (1 point)

2a) Find a basis for the solution space to homogeneous matrix equation $A\,\underline{x} = \underline{0}$, where A is the matrix on the left below, and its reduced row echelon form is on the right.

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

$$\left[\begin{array}{ccccc|c} 2 & 6 & 0 & -6 & 1 & 0 \\ 0 & 0 & 3 & 6 & 7 & 0 \\ 3 & 9 & -8 & -25 & 5 & 0 \\ -1 & -3 & 4 & 11 & 6 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccccc|c} 1 & 3 & 0 & -3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(6 points)

backsolve $x_5 = 0$
$x_4 = t$
$x_3 = -2t$
$x_2 = s$
$x_1 = -3s + 3t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3s+3t \\ s \\ -2t \\ t \\ 0 \end{bmatrix} = s\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\text{basis: } \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}.}$$

2b) What is the dimension of the solution space in part 2a?

(1 point)

$$\boxed{2.}$$

3) Consider the differential equation below for a function $x(t)$, which could arise from an unforced mass-spring configuration:

$$x''(t) + 2x'(t) + 10x(t) = 0.$$

3a) Find the general solution to this homogeneous linear differential equation. Hint: use the characteristic polynomial method to first find a basis.

(6 points)

$p(r) = r^2 + 2r + 10 = 0$
$(r+1)^2 + 9 = 0$
$(r+1)^2 = -9$
$r + 1 = \pm 3i$
$r = -1 \pm 3i$

complex solns $e^{(-1+3i)t}$, $e^{(-1-3i)t}$

real basis $e^{-t}\cos 3t$, $e^{-t}\sin 3t$

$$\boxed{x(t) = c_1 e^{-t}\cos 3t + c_2 e^{-t}\sin 3t}$$

3b) Which of the three damping phenomena is exhibited by solutions to this differential equation?

(1 point)

underdamped (complex roots to $p(r)$).

3c) Now consider the inhomogeneous DE

$$x''(t) + 2x'(t) + 10x(t) = -20.$$

Notice that $x_p(t) = -2$ is a particular solution. Use this fact and your previous work to write down the general solution to the inhomogeneous DE.

(2 points)

$x = x_p + x_H$
$= -2 + c_1 e^{-t}\cos 3t + c_2 e^{-t}\sin 3t$

3d) Solve the initial value problem

$$x''(t) + 2x'(t) + 10x(t) = -20$$
$$x(0) = 1$$
$$x'(0) = 3.$$

(6 points)

$x(t) = -2 + c_1 e^{-t}\cos 3t + c_2 e^{-t}\sin 3t$
$x'(t) = c_1(-e^{-t}\cos 3t - 3e^{-t}\sin 3t) + c_2(-e^{-t}\sin 3t + 3e^{-t}\cos 3t)$

$x(0) = 1 = -2 + c_1 \Rightarrow c_1 = 3$
$x'(0) = 3 = -c_1 + 3c_2 \Rightarrow 3 = -3 + 3c_2$
$6 = 3c_2 \Rightarrow c_2 = 2$

$$\boxed{x(t) = -2 + 3e^{-t}\cos 3t + 2e^{-t}\sin 3t}$$