

# Math 2250    Week 4 Super Quiz

Name, UID, and section TA: \_\_\_\_\_

**Write your answer in the space provided. Show work for full credit.**

1. Consider the following differential equation for  $y(x)$ :

$$y'(x) = 4xy - 8x.$$

- (a) (5 points) Find the general solution to this DE, using the algorithm for separable differential equations

- (b) (5 points) Find the general solution to this DE, using the algorithm for linear differential equations

2. Consider the following linear drag initial value problem:

$$v'(t) = 6 - 0.2v$$

$$v(0) = 0$$

(a) (3 points) Use a phase diagram to determine the limiting velocity as  $t \rightarrow \infty$  for the solution to this IVP.

(b) (4 points) Solve the initial value problem above. (Your solution should be consistent with your phase diagram in part (a).)

(c) (3 points) Convert the following information into a differential equation initial value problem for the velocity  $v(t)$  of a boat moving in a straight line on a lake. Your IVP should end up being equivalent to the IVP at the top of the page: A small motor boat and its pilot weigh a total of 640 pounds. The motor provides a thrust force of 120 pounds. The drag from the boat depends linearly on the velocity  $v$  of the boat, and is 4 pounds for each  $\frac{ft}{sec}$  of velocity. Aside from the motor and drag forces, there are no other forces acting on the horizontal motion of the boat, and the boat starts from rest. Hint: recall that in the English system, every 32 pounds of force corresponds to 1 slug of mass. (fyi, we use this conversion factor although it is not exact; force in pounds and mass in slugs are related by the gravitational identity  $F = mg$ , and the acceleration of gravity is not exactly  $32 \frac{ft}{s^2}$ .)

3. (5 points) Consider a brine tank containing 300 gallons of initially pure water, and with input and output pipes. At time  $t = 0$  the system is turned on, with water flow rates of 60 gallons per hour into and out of the tank. The water flowing into the tank is a brine solution, with 0.1 pounds of salt per gallon of brine. Derive the initial value problem for the amount  $x(t)$  of salt in the tank at time  $t$ . It should be equivalent to the IVP in problem 2. (Since you solved the IVP in problem 2, there is no need to resolve it.)