Math 2250-4
Tues Sept 3
We will mostly use last Friday's notes. Our goals today are

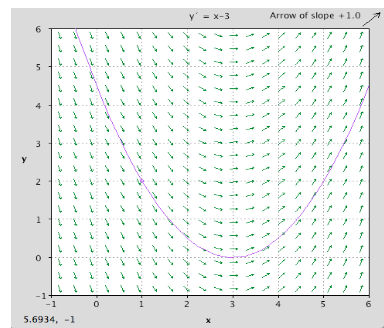
(1) understand what makes a first order differential equation separable.
(2) understand the algorithm based on differentials that solves separable differential equations: why it works, and how it sometimes misses "singular solutions"
(3) understand and apply the existence-uniqueness theorem for first order DE initial value problems.

When discussing the existence-uniqueness theorem on Friday's notes today, there is a reference back to two initial value problems from Wednesday's notes. Those were:

Exercise 4 (last Wednesday):

$$\frac{dy}{dx} = x - 3$$
$$y(1) = 2\,.$$

We found the solution $y(x) = \frac{x^2}{2} - 3\,x + \frac{9}{2} = \frac{(x-3)^2}{2}$. Is this consistent with the existence-uniqueness theorem?



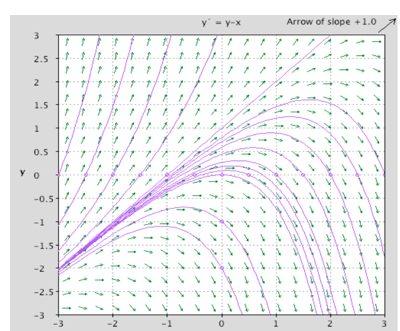
Exercise 5: (last Wednesday)

$$\frac{dy}{dx} = y - x$$
$$y(0) = 0$$

From a family of solutions that was given to us, we found a solution

$$y(x) = x + 1 - e^{x}\,.$$

Is this the only possible solution? Hint: use the existence-uniqueness theorem.



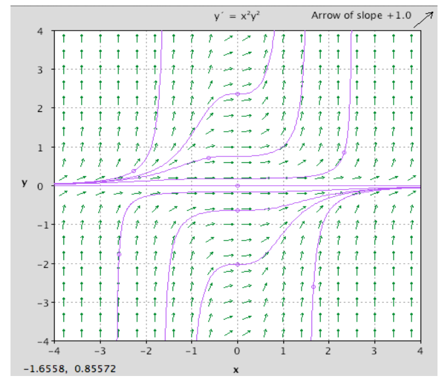
Exercise 1 (today): Here's another example of using a separable DE to illustrate the existence-uniqueness theorem.

a) Does each IVP

$$y' = x^2\, y^2$$
$$y(x_0) = y_0$$

have a unique solution?

b) Find all solutions to this differential equation.



Maple check (notice it misses the singular solution):

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> with(DEtools) :
  dsolve(y'(x) = x^2·y(x)^2, y(x));
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$$y(x) = \frac{3}{-x^3 + 3\ \_C1} \tag{1}$$

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Exercise 2: Do the initial value problems below always have unique solutions? Can you find them? (Notice these are NOT separable differential equations.) Can Maple find formulas for the solution functions?

a)

$$y' = x^2 + y^2$$
$$y(x_0) = y_0$$

b)

$$y' = x^4 + y^4$$
$$y(x_0) = y_0$$