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Quiz 7
November 8, 2013
SOLUTIONS

1) Consider the following four mechanical oscillation differential equations. In each case answer the following questions:

(i) If the problem is inhomogeneous what is the undetermined coefficients "guess" for the particular solution $x_P(t)$? (Do NOT try to find the precise particular solution, just its form.) If the problem is homogenous, find the homogeneous solution.
(ii) What physical phenomenon will be exhibited by the general solutions to this differential equation?

<u>1a)</u> $x''(t) + 10\,x(t) = 0$

(2 points)

(i) (1 point) This is homogeneous, and the characteristic polynomial is $p(r) = r^2 + 10$. It has roots $r = \pm\, i\sqrt{10}$ so the solution homogeneous solutions are of the form

$$x(t) = A\cos\left(\sqrt{10}\,t\right) + B\sin\left(\sqrt{10}\,t\right)$$

(ii) (1 point) <u>Simple harmonic motion</u> ($x(t) = C\cos\left(\omega_0 t - \alpha\right)$).

<u>1b)</u> $x''(t) + 9\,x(t) = 4\cos(3\,t)$.

(3 points)

(i) (2 points)

$$x_P(t) = t\left(d_1\cos(3\,t) + d_2\sin(3\,t)\right) = d_1 t\cos(3\,t) + d_2 t\sin(3\,t)\ .$$

(Reason: Since the characteristic polynomial for the homogeneous problem is $p(r) = r^2 + 9 = (r + 3\,i)(r - 3\,i)$ and since the forcing function $4\cos(3\,t)$ is one of the real basis functions associated to the exponential solutions $e^{\pm 3\,i\,t}$ our undetermined coefficients particular solution has the "extended case" form)

(ii) (1 point) This is an undamped system forced at the natural angular frequency, so solutions will display <u>resonance</u>.

<u>1c)</u> $x''(t) + 10\,x(t) = 4\cos(3\,t)$.

(2 points)

(i) (1 point) $x_P(t) = A\cos(3\,t) + B\sin(3\,t)$, or $x_P(t) = A\cos(3\,t)$ (since "L" only has even derivatives).

(ii) (1 point) <u>Beating</u>, since the DE is undamped $\omega = 3$ is close to $\omega_0 = \sqrt{10}$.

<u>1d)</u> $x''(t) + 0.2\,x'(t) + 9\,x(t) = 4\cos\left(\omega\,t\right)$,
with $\omega \approx 3$.

(3 points)

(i) (1 point) $x_p(t) = A\cos(\omega t) + B\sin(\omega t)$

(ii) (2 points) Since the damping is small the undetermined coefficients particular solution will be the steady periodic solution, and these solutions will demonstrate practical resonance when $\omega \approx 3$.