Name + UID:________________________ Section (circle one): C.Wei S.Bagley

Math 2250-4
Quiz 6
October 25, 2013

1a) Consider the differential equation for $y(x)$

$$y''(x) + 3\,y'(x) - 4\,y(x) = 0\,.$$

Find the general solution to this homogeneous differential equation. Hint: the solution space has a basis consisting of exponential functions.

(5 points)

If we try $y(x) = e^{r\,x}$ we compute

$$L\left(e^{r\,x}\right) = r^2 e^{r\,x} + 3\,r\,e^{r\,x} - 4\,e^{r\,x} = e^{r\,x}\left(r^2 + 3\,r - 4\right).$$

Since we want $L\left(e^{r\,x}\right) = 0$ *the exponential growth rates r must be roots of the characteristic polynomial, i. e they must solve*

$$0 = r^2 + 3\,r - 4 = (r+4)(r-1)\,.$$

Since the roots are $r = -4, 1$, $y_1(x) = e^{-4\,x}$, $y_2(x) = e^x$ *solve the DE, and the general homogeneous solution is*

$$y_H(x) = c_1 e^{-4\,x} + c_2 e^x.$$

1b) Verify that $y(x) = -2$ is a solution to the inhomogeneous differential equation

$$y''(x) + 3\,y'(x) - 4\,y(x) = 8.$$

(1 points)

for $y(x) = -2$, $y''(x) + 3\,y'(x) - 4\,y(x) = 0 + 0 - 4(-2) = 8$, *so the DE is true for this constant function and it is a (particular) solution.*

1c) Combine your work from <u>a</u>, <u>b</u> to deduce the general solution to the inhomogeneous DE in <u>b.</u> Then use this general solution to solve the initial value problem

$$y''(x) + 3\,y'(x) - 4\,y(x) = 8$$
$$y(0) = 0$$
$$y'(0) = 2.$$

(4 points)

The general solution is

$$y(x) = y_P(x) + y_H(x) = -2 + c_1 e^{-4\,x} + c_2 e^x.$$

So

$$y'(x) = y_P'(x) + y_H'(x) = 0 - 4\,c_1 e^{-4\,x} + c_2 e^x\,.$$

To solve the IVP we set x = 0 and solve for the free parameters:

$$y(0) = 0 = -2 + c_1 + c_2$$
$$y'(0) = 2 = -4\,c_1 + c_2.$$
$$c_1 + c_2 = 2$$
$$-4\,c_1 + c_2 = 2.$$

Subtracting equations implies

$c_1 = 0$, so also $c_2 = 2$, and the solution is

$$y(x) = -2 + 2\,e^x .$$

```
> with(DEtools):
  dsolve({y''(x) + 3·y'(x) − 4·y(x) = 8, y(0) = 0, y'(0) = 2}); #Maple check
```

$$y(x) = -2 + 2\,\mathrm{e}^x \tag{1}$$

```
>
```