Math 2250-4 Quiz 6 October 25, 2013

1a) Consider the differential equation for y(x)

$$y''(x) + 3y'(x) - 4y(x) = 0$$
.

Find the general solution to this homogeneous differential equation. Hint: the solution space has a basis consisting of exponential functions.

(5 points)

If we try
$$y(x) = e^{rx}$$
 we compute
 $L(e^{rx}) = r^2 e^{rx} + 3r e^{rx} - 4e^{rx} = e^{rx}(r^2 + 3r - 4).$

Since we want $L(e^{rx}) = 0$ the exponential growth rates r must be roots of the characteristic polynomial, *i*. *e they must solve*

 $0 = r^2 + 3r - 4 = (r + 4)(r - 1).$ Since the roots are r = -4, 1, $y_1(x) = e^{-4x}$, $y_2(x) = e^x$ solve the DE, and the general homogeneous solution is

$$y_H(x) = c_1 e^{-4x} + c_2 e^x.$$

1b) Verify that y(x) = -2 is a solution to the inhomogeneous differential equation y''(x) + 3y'(x) - 4y(x) = 8.

(1 points)

for y(x) = -2, y''(x) + 3y'(x) - 4y(x) = 0 + 0 - 4(-2) = 8, so the DE is true for this constant function and it is a (particular) solution.

1c) Combine your work from <u>a</u>, <u>b</u> to deduce the general solution to the inhomogeneous DE in <u>b</u>. Then use this general solution to solve the initial value problem

$$y''(x) + 3y'(x) - 4y(x) = 8$$

 $y(0) = 0$
 $y'(0) = 2.$

(4 points)

The general solution is

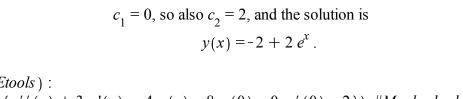
$$y(x) = y_P(x) + y_H(x) = -2 + c_1 e^{-4x} + c_2 e^{x}$$

So

$$y'(x) = y_{p'}(x) + y_{H'}(x) = 0 - 4c_{1}e^{-4x} + c_{2}e^{x}$$

To solve the IVP we set $x = 0$ and solve for the free parameters:
 $y(0) = 0 = -2 + c_{1} + c_{2}$
 $y'(0) = 2 = -4c_{1} + c_{2}$.
 $c_{1} + c_{2} = 2$
 $-4c_{1} + c_{2} = 2$.

Subtracting equations implies



> with (DEtools):
dsolve(
$$\{y''(x) + 3 \cdot y'(x) - 4 \cdot y(x) = 8, y(0) = 0, y'(0) = 2\}$$
); #Maple check
 $y(x) = -2 + 2 e^{x}$
(1)