Name + UID:________________________ Section (circle one): C.Wei S.Bagley

Math 2250-4
Quiz 5
October 11, 2013

1a) Define what a <u>linear combination</u> of vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ is.

(2 points)

any vector that is a sum of multiples of those vectors, i.e. any $\underline{v}$ *that can be expressed as*

$$\underline{v} = c_1 \underline{v}_1 + c_2\, \underline{v}_2 + \ldots + c_n\, \underline{v}_n\,.$$

1b) Define what it means for vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ to be <u>linearly independent.</u>

(2 points)

The only linear combination of $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ *that sums to zero is the "trivial" one with all* $c_i = 0$, *i.e.*

$$c_1 \underline{v}_1 + c_2\, \underline{v}_2 + \ldots + c_n\, \underline{v}_n = 0 \Rightarrow \boldsymbol{c}_1 = c_2 = \ldots = c_n = 0.$$

2) The span of the vectors

$$\underline{u} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

is a plane in $\mathbb{R}^3$. Find for which $[x, y, z]^T$ the system

$$c_1 \underline{u} + c_2 \underline{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

can be solved for c_1, c_2, in order to find the implicit equation $a\,x + b\,y + c\,z = d$ of this plane.

The augmented matrix for the linear combination equation

$$c_1 \underline{u} + c_2 \underline{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

is

$$\left[\begin{array}{cc|c} 1 & 2 & x \\ -2 & -3 & y \\ 0 & 4 & z \end{array}\right].$$

Reduce it:

$2\,R_1 + R_2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 2 & x \\ 0 & 1 & 2\,x + y \\ 0 & 4 & z \end{array}\right]$$

$-4\,R_1 + R_3 \rightarrow R_3$:

$$\left[\begin{array}{cc|c} 1 & 2 & x \\ 0 & 1 & 2x + y \\ 0 & 0 & -8x - 4y + z \end{array}\right].$$

So the system for c_1, c_2 is consistent if and only if $[x, y, z]^T$ lies on the plane with implicit equation

$$-8x - 4y + z = 0.$$

(6 points)