Math 2250-4
Quiz 4 SOLUTUONS
September 27, 2013

1a) *Consider the following system of equations*

$$\begin{aligned} 2x - 3y - z &= 0 \\ -2x + y \quad &= 2 \\ -x + y + z &= 2 \end{aligned}$$

Exhibit the augmented matrix corresponding to this system, compute its reduced row echelon form, and find the solution set to the system.

(5 points)

$$\left[\begin{array}{ccc|c} 2 & -3 & -1 & 0 \\ -2 & 1 & 0 & 2 \\ -1 & 1 & 1 & 2 \end{array}\right]$$

swap first and third rows:

$$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 2 \\ -2 & 1 & 0 & 2 \\ 2 & -3 & -1 & 0 \end{array}\right]$$

$-R_1 \rightarrow R_1$:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ -2 & 1 & 0 & 2 \\ 2 & -3 & -1 & 0 \end{array}\right]$$

$2R_1 + R_2 \rightarrow R_2$; $-2R_1 + R_3 \rightarrow R_3$:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ 0 & -1 & -2 & -2 \\ 0 & -1 & 1 & 4 \end{array}\right]$$

$-R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 1 & 4 \end{array}\right]$$

$R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 3 & 6 \end{array}\right].$$

$\frac{R_3}{3} \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$-2R_3 + R_2 \rightarrow R_2$; $R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

Thus the solution is the single point

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}.$$

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> with(LinearAlgebra) : #Maple check
  ReducedRowEchelonForm([[2, -3, -1, 0], [-2, 1, 0, 2], [-1, 1, 1, 2]]);
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$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad (1)$$

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1b) *Consider other linear systems $A\,\underline{\mathbf{x}} = \underline{\mathbf{b}}$ that have the same coefficient matrix A as in part **1a**. What can you say about the solution sets to those systems, even if you are not told what the vector $\underline{\mathbf{b}}$ is? Explain*
(2 points)

The solution set will always be a single point. Because A reduces to the identity matrix, if we start with any augmented matrix $\langle A|\underline{\mathbf{b}}\rangle$ it will reduce to $\langle I|\underline{\mathbf{c}}\rangle$ for some vector $\underline{\mathbf{c}}$. The final augmented matrix represents the system of equation $x = c_1, y = c_2, z = c_3$.

2) *Consider the two matrices*

$$A := \begin{bmatrix} 2 & -3 & -1 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \; B := \begin{bmatrix} 2 & -2 & 0 \\ -3 & 0 & 4 \end{bmatrix}.$$

Only one of the two products AB, BA is defined. Which is it and why? Then compute that product matrix.
(3 points)

In any matrix product, the number of columns of the matrix on the left must equal the number of rows of the matrix on the right. Thus $A_{3\times3}B_{2\times3}$ is not defined, but $B_{2\times3}A_{3\times3} = (BA)_{2\times3}$ is.

$$\begin{bmatrix} 2 & -2 & 0 \\ -3 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -1 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -8 & -2 \\ -10 & 13 & 7 \end{bmatrix}.$$

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> [[2, -2, 0], [-3, 0, 4]] . [[2, -3, -1], [-2, 1, 0], [-1, 1, 1]]; #that's a period as the multiplication operator
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$$\begin{bmatrix} 8 & -8 & -2 \\ -10 & 13 & 7 \end{bmatrix} \quad (2)$$

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