Math 2250 Week 2 Quiz

Name, UID, and section number: _____

Write your answer in the space provided. Show work for full credit.

1. (3 points) Consider the differential equation for $y(x)$:

$$y' = -3y + 9.$$

Is this differential equation separable, linear, both separable and linear, or neither separable nor linear? Explain.

Solution: Both: The equation is separable, because dy/dx is a product of a function of x times a function of y, for example (taking the function of x to be the constant -3):

$$\frac{dy}{dx} = -3(y-3)$$

The differential equation is linear because we can rewrite it in the form

$$y' + P(x)y = Q(x),$$

namely

$$y' + 3y = 9.$$

2. (7 points) Solve the initial value problem

$$y' = -3y + 9$$

$$y(0) = 5$$

Solution: Using the separable DE algorithm:

$$\frac{dy}{dx} = -3(y-3) \Rightarrow \frac{dy}{y-3} = -3dx \Rightarrow \int \frac{dy}{y-3} = \int -3dx$$

$$\Rightarrow ln(|y-3|) = -3x + C \Rightarrow |y-3| = e^{-3x+C} = e^{-3x}e^{C}$$
$$\Rightarrow y - 3 = C_1 e^{-3x},$$

where C_1 is either $+e^C$ or $-e^C$. Thus $y(x) = 3 + Ce^{-3x}$. $y(0) = 5 \Rightarrow C = 2$, and

$$y(x) = 3 + 2e^{-3x}.$$

Alternately, using the linear DE algorithm,

$$y' + 3y = 9.$$

Since $P(x) = 3$, we choose integrating factor $e^{\int 3dx} = e^{3x}$:

$$e^{3x}(y' + 3y) = 9e^{3x} \Rightarrow \frac{d}{dx}(e^{3x}y) = 9e^{3x}$$

$$\Rightarrow e^{3x}y = \int 9e^{3x}dx = 3e^{3x} + C.$$

Dividing by the exponential e^{3x} (i.e. multiplying by e^{-3x}) yields

$$y = 3 + Ce^{-3x}$$

as before. And substituting $y(0) = 5$ implies $C = 2$ as before.