Math 2250 Week 1 Quiz

Name, UID, and section number: _____

Write your answer in the space provided. Show work for full credit.

1. (10 points) Verify that for every constant C, the functions $y(x) = -2+Ce^{4x}$ are solutions to the following differential equation:

$$y' - 4y = 8.$$

Solution: Find y' for $y(x) = -2 + Ce^{4x}$:

$$y'(x) = 4Ce^{4x}.$$

Plug in:

$$y' - 4y = 4Ce^{4x} - 4(-2 + Ce^{4x}) = 4Ce^{4x} + 8 - 4Ce^{4x} = 8.$$

So, the functions $y(x)$ make the differential equation true; so they are solutions.

2. (10 points) A object moves along a number line, with position function $x(t)$ m at time t. This object is subject to an acceleration of $a(t) = 8sin(2t)$ $\frac{m}{s^2}$. Its initial position and velocity are $x_0 = 0$ m, $v_0 = 0$ $\frac{m}{s}$. Find the position function $x(t)$.

Solution: Integrate the acceleration to find the velocity; then integrate the velocity to find the position:

$$v'(t) = a(t) = 8sin(2t) \Rightarrow v(t) = -4cos(2t) + C.$$

Since $v_0 = 0$, $0 = -4 + C$. Thus $C = 4$ and $v(t) = -4cos(2t) + 4$. Integrating once more we find $x(t)$:

$$x'(t) = -4cos(2t) + 4 \Rightarrow x(t) = -2sin(2t) + 4t + C.$$

$x_0 = 0 \Rightarrow 0 = 0 + 0 + C \Rightarrow C = 0$, so $x(t) = -2sin(2t) + 4t\ m$.