Math 2250-4

Friday, December 13

Course review

Final exam: Tuesday December 17, 8:00 a.m. -10:00 a.m. (I will let you work until 10:15). This is the official University time and location - our lecture room WEB L104 (here). As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. The algebra and math on the exam should all be doable by hand.

Review of previous final exam: (today) Friday December 13, 3:30-5:30 p.m., here in WEB L104.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters. Also consult the “course learning objectives” in our syllabus.

Chapters

 1-2: 10-20% first order DEs

 3-4: 20-30% matrix algebra and vector spaces

 5, EP3.7: 15-30% linear differential equations and applications

 6.1-6.2: 15-30% eigenvalues and eigenvectors, including complex case

 7.1-7.4: 20-40% linear systems of differential equations and applications

 9.1-9.4: 15-20% non-linear autonomous systems of DEs and applications

 10.4-10.5, EP 7.6: 15-30% Laplace transform

On the next page is a more detailed list of the topics we’ve investigated this semester. They are more inter-related than you may have realized at the time, so let’s discuss the connections. Then we’ll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

 slope fields, Euler approximation

 phase diagrams for autonomous DEs

 equilibrium solutions

 stability

 existence-uniqueness thm for IVPs

 methods:

 separable

 linear

 applications

 populations

 velocity-acceleration models

 input-output models

3-4 matrix algebra and vector spaces

 linear systems and matrices

 reduced row echelon form

 matrix and vector algebra

 manipulating and solving matrix-

 vector equations for unknown

 vectors or matrices.

 matrix inverses

 determinants

 vector space concepts

 vector spaces and subspaces

 linear combinations

 linear dependence/independence

 span

 basis and dimension

 linear transformations

 aka superposition

 fundamental theorem for solution

 space to L(y)=f when L is linear

5 Linear differential equations

 IVP existence and uniqueness

 Linear DEs

 Homogeneous solution space,

 its dimension, and why

 superposition, x(t)= xP+ xH

 Constant coefficient linear DEs

 xH  via characteristic polynomial

 Euler’s formula, complex roots

 xP  via undetermined coefficients

 solving IVPs

 applications:

 mechanical configurations

 unforced: undamped and damped

 cos and sin addition angle formulas

 and amplitude-phase form

 forced undamped: beating, resonance

 forced damped: **x**sp+ **x**tr, practical

 resonance

 RLC circuits

 Using conservation of total energy

 (=KE+PE) to derive equations of

 motion, especially for mass-spring and

 pendulum

6.1-6.1 eigenvalues, eigenvectors (eigenspaces), diagonalizable matrices; real+complex eigendata.

7.1-7.4 linear systems of DEs

 first order systems of DEs and tangent vector

 fields.

 existence-uniqueness thm for first order IVPs

 superposition, **x**= **x**P+ **x**H

dimension of solution space for **x**H .

 conversion of DE IVPs or systems to first

 order system IVPs.

 Constant coefficient systems and methods:

 **x’**(t)=A**x**

 **x’**(t)=A**x**+**f**(t)

 **x’’**(t)=A**x** (from conservative systems)

 **x’’**(t)=A**x**+**f**(t)

 applications: phase portrait interpretation of unforced oscillation problems; input-output modeling; force and unforced mass-spring systems.

9.1-9.4 non-linear systems of differential equations

 autonomous systems of first order DEs

 equilibrium solutions

 stability

 phase portraits

 linearization near equilibria, stability analysis,

 further classification and qualitative

 sketching.

 Applications to interacting populations and

 non-linear mechanical configurations, esp.

 pendulum

10.1-10.5, EP7.6: Laplace transform

 definition, for direct computation

 using table for Laplace and inverse Laplace

 transforms … including for topics before/after

 the second midterm, i.e. on/off and impulse,

 forcing, convolution solutions

 Solving linear DE (or system of DE) IVPs with

 Laplace transform.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

x’’(t) + 5 x’(t) + 4 x(t) = 0 x’’(t) + 5 x’(t) + 4 x(t) = 3 cos(2t)