Math 2250-4

Friday, December 13

Course review

Final exam: Tuesday December 17, 8:00 a.m. -10:00 a.m. (I will let you work until 10:15). This is the official University time and location - our lecture room WEB L104 (here). As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. The algebra and math on the exam should all be doable by hand.

Review of previous final exam: (today) Friday December 13, 3:30-5:30 p.m., here in WEB L104.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters. Also consult the “course learning objectives” in our syllabus.

Chapters

1-2: 10-20% first order DEs

3-4: 20-30% matrix algebra and vector spaces

5, EP3.7: 15-30% linear differential equations and applications

6.1-6.2: 15-30% eigenvalues and eigenvectors, including complex case

7.1-7.4: 20-40% linear systems of differential equations and applications

9.1-9.4: 15-20% non-linear autonomous systems of DEs and applications

10.4-10.5, EP 7.6: 15-30% Laplace transform

On the next page is a more detailed list of the topics we’ve investigated this semester. They are more inter-related than you may have realized at the time, so let’s discuss the connections. Then we’ll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

slope fields, Euler approximation

phase diagrams for autonomous DEs

equilibrium solutions

stability

existence-uniqueness thm for IVPs

methods:

separable

linear

applications

populations

velocity-acceleration models

input-output models

3-4 matrix algebra and vector spaces

linear systems and matrices

reduced row echelon form

matrix and vector algebra

manipulating and solving matrix-

vector equations for unknown

vectors or matrices.

matrix inverses

determinants

vector space concepts

vector spaces and subspaces

linear combinations

linear dependence/independence

span

basis and dimension

linear transformations

aka superposition

fundamental theorem for solution

space to L(y)=f when L is linear

5 Linear differential equations

IVP existence and uniqueness

Linear DEs

Homogeneous solution space,

its dimension, and why

superposition, x(t)= xP+ xH

Constant coefficient linear DEs

xH  via characteristic polynomial

Euler’s formula, complex roots

xP  via undetermined coefficients

solving IVPs

applications:

mechanical configurations

unforced: undamped and damped

cos and sin addition angle formulas

and amplitude-phase form

forced undamped: beating, resonance

forced damped: **x**sp+ **x**tr, practical

resonance

RLC circuits

Using conservation of total energy

(=KE+PE) to derive equations of

motion, especially for mass-spring and

pendulum

6.1-6.1 eigenvalues, eigenvectors (eigenspaces), diagonalizable matrices; real+complex eigendata.

7.1-7.4 linear systems of DEs

first order systems of DEs and tangent vector

fields.

existence-uniqueness thm for first order IVPs

superposition, **x**= **x**P+ **x**H

dimension of solution space for **x**H .

conversion of DE IVPs or systems to first

order system IVPs.

Constant coefficient systems and methods:

**x’**(t)=A**x**

**x’**(t)=A**x**+**f**(t)

**x’’**(t)=A**x** (from conservative systems)

**x’’**(t)=A**x**+**f**(t)

applications: phase portrait interpretation of unforced oscillation problems; input-output modeling; force and unforced mass-spring systems.

9.1-9.4 non-linear systems of differential equations

autonomous systems of first order DEs

equilibrium solutions

stability

phase portraits

linearization near equilibria, stability analysis,

further classification and qualitative

sketching.

Applications to interacting populations and

non-linear mechanical configurations, esp.

pendulum

10.1-10.5, EP7.6: Laplace transform

definition, for direct computation

using table for Laplace and inverse Laplace

transforms … including for topics before/after

the second midterm, i.e. on/off and impulse,

forcing, convolution solutions

Solving linear DE (or system of DE) IVPs with

Laplace transform.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

x’’(t) + 5 x’(t) + 4 x(t) = 0 x’’(t) + 5 x’(t) + 4 x(t) = 3 cos(2t)