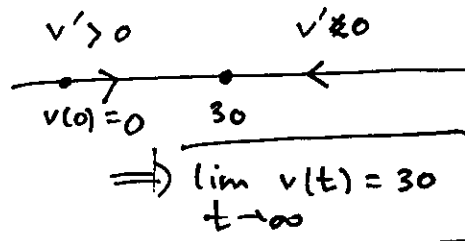Name_____

Student I.D._____

Math 2250-1
Quiz 4
September 14, 2012

1) Consider the following linear drag initial value problem:

$$\frac{dv}{dt} = 6 - 0.2\,v. \;=\; -.2\left(v - \frac{6}{.2}\right) = -.2\,(v - 30)$$

$$v(0) = 0.$$

1a) Use a phase diagram to determine the limiting velocity $\lim_{t \to \infty} v(t)$ for the solution to this IVP.

(2 points)

$$v' > 0 \qquad\qquad v' < 0$$

$$v(0) = 0 \qquad 30$$

$$\Rightarrow \lim_{t \to \infty} v(t) = 30$$

1b) Solve the initial value problem above. (Your solution should be consistent with the correct answer to part (a) above.)

(6 points)

$$v' + .2v = 6$$

$$e^{.2t}(v' + .2v) = 6e^{.2t}$$

$$(e^{.2t}v)' = 6e^{.2t}$$

$$e^{.2t}v = \int 6e^{.2t}\,dt = 30e^{.2t} + C$$

$$\div e^{.2t} \qquad v = 30 + Ce^{-.2t} \qquad v(0) = 0 \Rightarrow C = -30$$

$$\boxed{v(t) = 30 - 30e^{-.2t}}$$

so

1c) Use Newton's Law to convert the following information below into a differential equation initial value problem - your IVP should end up being equivalent to the IVP at the top of this page, if you keep track of your units correctly.

A small motor boat and its pilot weigh a total of 640 lbs. The motor provides a thrust force of 120 lbs. The drag from the boat depends on the velocity v of the boat, and is 4 lbs for each ft/sec of velocity v. Aside from the motor and the drag forces, there are no other forces acting on the boat, and the boat starts from rest. Hint: recall that in the English system every 32 pounds of force $= mg$ corresponds to 1 slug of mass.

(2 points)

$$w = 640\text{ lb} \qquad\qquad mx'' = F$$

$$m = \frac{640}{32} = 20.\text{ slug} \qquad 20v' = 120 - 4v$$

$$\boxed{\begin{array}{c} v' = 6 - \frac{1}{5}v \\[4pt] v(0) = 0 \end{array}}$$