Name Solutions.
Student I.D.

Math 2250-1
Quiz 3
September 7, 2012

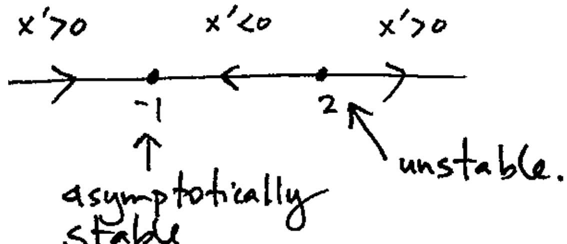
1) Consider the following differential equation for a function $x(t)$, which could be modeling a logistic population with harvesting:

$$x'(t) = 2(x-2)(x+1)\,.$$

1a) Draw a phase diagram. Identify the equilibrium solutions and whether or not they are stable.

equil sols
$x \equiv 2$
$x \equiv -1$.

(3 points)



1b) Solve the initial value problem for the differential equation above, with $x(0) = 0$:

$$x'(t) = 2(x-2)(x+1)$$
$$x(0) = 0\,.$$

Hint: To save time with partial fractions use the identity

$$\frac{1}{(x-a)(x-b)} = \frac{1}{a-b}\left(\frac{1}{x-a} - \frac{1}{x-b}\right).$$

(6 points)

$$\frac{dx}{(x-2)(x+1)} = 2\,dt$$

$$\frac{1}{3}\left(\frac{1}{x-2} - \frac{1}{x+1}\right)dx = 2\,dt$$

integrate: $\frac{1}{3}\ln\left|\frac{x-2}{x+1}\right| = 2t + C$

$\Rightarrow \ln\left|\frac{x-2}{x+1}\right| = 6t + C$

exp: $\left|\frac{x-2}{x+1}\right| = Ce^{6t}$

$\Rightarrow \frac{x-2}{x+1} = Ce^{6t}$

@ $t=0, x=0$

$\Rightarrow -2 = C$

$\frac{x-2}{x+1} = -2e^{6t}$

$x-2 = (x+1)(-2e^{6t}) = -2xe^{6t} - 2e^{6t}$

$x[1+2e^{6t}] = 2-2e^{6t}$

$$\boxed{x = \frac{2-2e^{6t}}{1+2e^{6t}}}$$

div by e^{6t} in num & denom

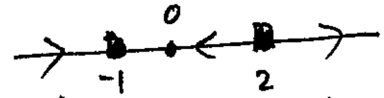
$$\boxed{x = \frac{2e^{-6t}-2}{e^{-6t}+2}}$$

1c) For your solution $x(t)$ to (b), verify that $\lim_{t\to\infty} x(t)$ does agree with the value implied by your phase diagram in part (a).

(1 point)

$$\lim_{t\to\infty} \frac{2e^{-6t}-2}{e^{-6t}+2} = \frac{0-2}{0+2} = -1\,.$$

since $-1 < \underset{x_0}{\overset{\shortparallel}{0}} < 2$,



phase diagram implies $x(t) \to -1$.