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Math 2250-1
Quiz 11 Solutions
November 21, 2012

1) Use the methods we've been discussing to find the general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

(10 points)

We try to find a basis for the solution space made of functions $e^{\lambda t}\underline{\boldsymbol{v}}$ where λ is an eigenvalue of the coefficient matrix, with eigenvector $\underline{\boldsymbol{v}}$. The characteristic polynomial is

$$p(\lambda) = \begin{vmatrix} 1-\lambda & 6 \\ 1 & 2-\lambda \end{vmatrix} = (\lambda-1)(\lambda-2) - 6 = \lambda^2 - 3\lambda - 4 = (\lambda+1)(\lambda-4).$$

So the eigenvalues are $\lambda = -1, 4$.

$\lambda = -1$; $[A - (-1)I]\underline{\boldsymbol{v}} = \underline{\boldsymbol{0}}$:

$$\left[\begin{array}{cc|c} 2 & 6 & 0 \\ 1 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right], \text{ so } \underline{\boldsymbol{v}} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \text{ is an eigenvector.}$$

$\lambda = 4$; $[A - 4I]\underline{\boldsymbol{v}} = \underline{\boldsymbol{0}}$:

$$\left[\begin{array}{cc|c} -3 & 6 & 0 \\ 1 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right], \text{ so } \underline{\boldsymbol{v}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ is an eigenvector.}$$

Thus the general solution is

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{-t} \begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$