Math 2250-1
Earthquake project
Due midnight Friday December 7, 2012

This project is optional. If you choose to complete it, it will be scored out of 20 points, and we will use this score to replace your lowest homework score. You may complete this project using whatever software you prefer - Maple, Matlab, Mathematica, Wolfram alpha, etc. You will submit your project work through CANVAS. If you complete it in Maple, Matlab or Mathematica you should submit your files with the output removed, but so that we can regenerate the output by executing your file(s). If you use on-line software, take screen shots of your commands and output, and paste these into an e.g. Microsoft Word document, along with an appropriate explanations.

Before you begin the project it is probably a good idea to review our class notes from Wednesday November 28. These notes include a discussion and Maple commands about how to model the ground-forced oscillation problem for a 3-story building.

The following directions are for Maple. Adapt as necessary if you're using other software.

0) Create a new document, and follow the same formatting directions as in the first project. Remember to only do mathematical computations inside of Math execution groups, in your work below.

You will be constructing a custom four story building to shake, based on your student number and directions I've slightly modified from the text on page 445:

For your personal four-story building to investigate, let the weight (in tons) of each story equal the largest digit of your student ID number and let k (in tons/ft) equal the smallest nonzero digit.

1) Do the appropriate change of units from tons of force to slugs of mass, and from tons/ft to lbs/ft as in the text version of this project, pages 444-445, to find and define the acceleration matrix $A_{4 \times 4}$ so that the vector $\underline{x}(t)$ of accelerations for each floor satisfies
$$\underline{x}''(t) = A\,\underline{x}$$
when there is no external forcing.

2) Find the eigendata for A and use this to make a table showing the natural angular frequencies (the omegas) and the corresponding natural periods. Make sure to leave the original Maple commands and output in your file, so we can check your work! Exhibit this data in a table like figure 7.4.17 on page 444, using decimal values with 5 digits for the angular frequencies and periods. (Maple can make tables. You may copy your table to a text section of your document, so that it is not deleted when you delete Maple output.) The class notes from November 28 contain commands that will help you clean up the data to make it clear that the eigenvalues and eigenvectors are real number decimals.

3) Use the eigendata information to explain the 4 fundamental modes of oscillation. For example, the slowest mode will have all 4 stories oscillating in phase, the so-called "sloshing mode". For each mode, give the angular frequency, explain which stories are oscillating in phase and out of phase with each other, and give the amplitude ratios for the various floors. We worked this out for the 3-story building on November 28.

4) Study practical resonance. We did a related but not identical problem on Nov. 28, using the class notes of Tuesday Nov. 27, pages 5-7. The first step is to use the method of undetermined coefficients to solve the forced oscillation problem

$$\underline{x}''(t) = A\underline{x} + E\omega^2 \cos(\omega t)\underline{b}$$

where $\underline{b} = [1, 1, 1, 1]^T$ and ω is assumed to not be one of the natural frequencies, and the shaking amplitude $E = 0.25$ ft, i.e. 3 inches. Try $x_P(t) = \cos(\omega t)\underline{c}$, solve for $\underline{c}$ symbolically, and then use Maple to define $\underline{c} = \underline{c}(\omega)$ with a formula that is similar to, but not exactly the same as the one on page 6 of the Nov. 26 notes. Do not have Maple display the very complicated and long formulas for $\underline{c}(\omega)$, although you might want to have a look at some intermediate step. (The formulas are a mess because Maple is using the adjoint formula for the inverse of a four by four matrix....this is actually not an efficient way to solve this system numerically.)

5) Create an approximate resonance picture like Figure 7.4.18 page 445 of text, for your building. It should have four peaks, not seven. Note that in this graph you are to compute the maximum amplitude of the **c**-vector as a function of period, not of angular frequency. The Maple LinearAlgebra command that computes the maximum absolute value of the entries of a vector $\underline{v}$ is $Norm(\underline{v})$, whereas in the Nov. 27 notes we computed the Euclidean magnitude of the entire vector, which was $Norm(\underline{v}, 2)$.

6) Is your particular building susceptible to likely damage from an earthquake having its period in the 2 to 3 second range? Explain.