Math 2250-1
Week 5 concepts and homework, due September 21.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

3.1-3.3: linear systems of (algebraic) equations; augmented matrices and Gaussian elimination to compute reduced row-echelon form. Explicitly specifying the solution space by backsolving the reduced row echelon form of the augmented matrix. Depending on your previous experience and background, you may need to practice this algebra a lot, or just a little. So, I've included a fairly large number of good practice problems in addition to the ones you'll hand in.

3.1: 1, **4**, **6**, 11, **16**, 17, 19, 23, 24, 27, **28**, 29, 32, 33, **34**
3.2: 7, **8**, 9, 13, 17, **20** (backsolve from row echelon form), 29, 30;
3.3: 13, 19, 33, **34**;

3.4: matrix operations and algebra

3.4: 3, 5, 7, **10,** 13**,** **16**, 19, 27, 31, **32**, **34**, 39, **40**, 44.

w5.1 Consider the augmented matrix for 3.2.20 above:

$$\left[\begin{array}{rrrrr|r} 2 & 4 & -1 & -2 & 2 & 6 \\ 1 & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{array}\right].$$

a) Continue your work from **3.2.20** to compute the reduced row echelon form of this augmented matrix, and verify that you recover the same explicit solutions as you did in that problem, when you backsolved from a row echelon form.
b) Use technology to recompute the reduced row echelon form, which you've already computed by hand in **a**. (If you don't want to use Maple, Matlab, or your graphing calculator, google will quickly find lots of internet sites to do this for you. Hand in some evidence of your technology work.
c) The vertical bar in the augmented matrix really has nothing to do with its reduced row echelon form. If we think of putting the vertical bar in different places or just focusing on certain columns, we can answer questions about related linear systems of equations. For example, use the reduced row echelon form the entire augmented matrix to write down the solution $[x_1, x_2, x_3]^T$ to the linear system

$$\begin{aligned} 2x_1 + 4x_2 - x_3 &= -2 \\ x_1 + 3x_2 + 2x_3 &= -7 \\ 5x_1 + 8x_2 - 7x_3 &= 6. \end{aligned}$$

(Notice how the augmented matrix for this problem is related to parts of the original augmented matrix.)
d) Use the same reduced row echelon form computation to write down the solution $[x_1, x_2, x_3]^T$ to

$$\begin{aligned} 2x_1 + 4x_2 - x_3 &= 2 \\ x_1 + 3x_2 + 2x_3 &= 3 \\ 5x_1 + 8x_2 - 7x_3 &= 1. \end{aligned}$$

e) Use the same computation to discuss the solution set of $[x_1, x_2]^T$ satisfying the system

$$\begin{aligned} 2x_1 + 4x_2 &= -1 \\ x_1 + 3x_2 &= 2 \\ 5x_1 + 8x_2 &= -7. \end{aligned}$$

w5.2 Consider the matrix

$$A := \begin{bmatrix} 0 & 3 & 6 & 1 & 4 & 13 \\ 0 & 5 & 10 & 8 & 13 & 47 \\ 0 & 2 & 4 & 5 & 7 & 26 \end{bmatrix}.$$

a) Compute the reduced row echelon form by hand.
b) Check your answer with technology.
c) Find all solutions to the homogeneous matrix equation $A\,\underline{\boldsymbol{x}} = \underline{\mathbf{0}}$, using your reduced row echelon form computation.
d) Check your answer to **c** with technology.