Math 2250-1

Week 4 concepts and homework, due September 14.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems. You will also have a Maple/Matlab project due on Monday September 17.

2.2: equilibria, stability, phase diagram analysis:

23 *(this problem was postponed from last week).*

2.3: improved velocity-acceleration models:

2, *3, 9,* **10, 12**: *constant, or constant plus linear drag forcing*

13, 14, 17, *18: quadratic drag*

25, 26: *escape velocity*

2.4-2.6: *numerical methods for approximating solutions to first order initial value problems. Your second Maple/Matlab project addresses these sections too.*

2.4: **4**: *Euler's method*

2.5: **4**: *improved Euler*

2.6: **4**: *Runge-Kutta*

w4.1) Runge-Kutta is based on Simpson's rule for numerical integration. Simpson's rule is based on the fact that for a subinterval of length $2\,h$, which by translation we may assume is the interval $-h \le x \le h$, the parabola $y = p(x)$ which passes through the points $(-h, y_0)$, $(0, y_1)$, (h, y_2) has integral

$$\int_{-h}^{h} p(x)\,\mathrm{d}x = \frac{2\,h}{6} \cdot \left(y_0 + 4\,y_1 + y_2\right). \qquad (1)$$

If we write the quadratic interpolant function $p(x)$ whose graph is this parabola as

$$p(x) = a + b\,x + c\,x^2$$

with unknown parameters a, b, c then since we want $p(0) = y_1$ we solve $y_1 = p(0) = a + 0 + 0$ to deduce that $a = y_1$.

w4.1a) Use the requirement that the graph of $p(x)$ is also to pass through the other two points, $(-h, y_0)$, (h, y_2) to express b, c in terms of h, y_0, y_1, y_2.

w4.1b) Compute $\int_{-h}^{h} p(x)\,\mathrm{d}x$ for these values of a, b, c and verify equation (1) above.