

Math 2250-1
Week 2 concepts and homework, due August 31.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

You should also try the software Maple this week. There are documents on our homework page that you can work through. We will run optional introductory Maple sessions in LCB 115.

1.3:

3, 6, 9: *geometric meaning of how slope fields relate to graphs of solutions to DE's and to IVP's*
11, 12, 13, 14: *understanding the existence-uniqueness theorem for solutions to IVP's*

(study section 1.4 separable DE's before doing these):

w2.1) In problem 1.3.13 above, the existence-uniqueness theorem guarantees a unique solution to the given initial value problem, on some interval containing $x = 0$.

w2.1a) Use separation of variables to find this solution.

w2.1b) What is the largest interval on which this solution exists? Hint: if you solved this problem correctly, the formula for $y(x)$ involves a square root.

w2.1c) Can you think of a way to extend your function in w2.1b, using a piecewise definition, in order to get a solution to the DE IVP that is defined for all $-\infty < x < \infty$? Is it the unique such solution to the IVP, defined for all x ? Hint: consider the differential equation solution that separation of variables missed.

1.4:

2, 3, 4, 9, 13, 19, 21: *solving DE's and IVP's for separable differential equations*
41, 42, 45, 46, 49, 60: *modeling and solving problems with first order separable DEs. Note: The correct answer to 42 is 4.25 billion years, not the 1.25 billion years that appears in the back of the book.*

1.5:

1, 7, 8, 13, 24: *solving linear DE's and IVP's.*
34, 36, 38, 41: *modeling and solving problems with first order linear DE's.*

w2.3) Consider the differential equation for $y = y(x)$:

$$y' = x + 2y + 1 + 2xy.$$

w2.3a) Although it is not immediately clear, this is a separable differential equation. Write it in separable form, and solve it using section 1.4 techniques. Hint: the right side factors.

w2.3b) This differential equation is also linear. Write it in linear form, and solve it using section 1.5 techniques.

w2.3c) Solve the initial value problem

$$\begin{aligned} y' &= x + 2y + 1 + 2xy \\ y(0) &= 1 \end{aligned}$$

By the way, you can check your answer to the IVP above by using Maple, via the commands

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> with(DEtools) : #loads differential equations package
[ dsolve( {y'(x) = x + 2·y(x) + 1 + 2·x·y(x), y(0) = 1 } );
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