

Math 2250-1

Week 1 concepts and homework, due Friday August 24 (at the start of class).

All of the indicated problems are good for seeing if you can work with the underlying concepts. The underlined problems are to be handed in. The quiz at the start of class on Friday will be drawn from all of these concepts and from these or related problems.

*1.1 Check whether a given function solves a **differential equation**. If solutions to a first order DE are given with a "free" constant C , find which C value solves a given **initial value problem**; translate geometric or modeling properties described in words into differential equations satisfied by the solution functions. Combine the above ideas to solve more complicated problems.*

1.1: 4, 5, 6, 9, 15, 19, 29, 30, 32, 33, 34, 35, 43, 45, 46 (use 43).

w1.1) Consider the first order differential equation for functions $y(x)$:

$$y' = 3y - 3x^2.$$

a) Show that functions $y(x) = x^2 + \frac{2}{3}x + \frac{2}{9} + C e^{3x}$ solve this DE, where C is any constant.

b) Solve the initial value problem for this DE, with $y(0) = -1$.

1.2 Differential equations $y'(x) = f(x)$ which can be solved by direct antidifferentiation

$y(x) = \int f(x)dx + C$: Solve such DE's using Calculus techniques. Solve for particle velocity and position, given a formula for the acceleration function. Solve for position if velocity is described graphically. Applications.

1.2: 1, 5, 6, 8, 10, 13, 15, 18, 19, 25, 26, 30, 36, 40.

w1.2) Solve the following initial value problems as a way to review important integration techniques from Calculus: substitution and integration by parts.

a) $\frac{dy}{dx} = e^{2x}$, $y(0) = 1$.

b) $y'(x) = x \sin(3x)$, $y(0) = 0$.

c) $\frac{dy}{dx} = \frac{3x}{\sqrt{x^2 + 4}}$, $y(0) = 2$.

w1.3 This is like problem 1.2.19. Consider a car that travels for 5 minutes along a road, starting and ending at rest. Its velocity function has a graph made of line segments, as in problems 19-22. The horizontal units are minutes, and the vertical units are miles/minute. The three line segments in the graph of the velocity function connect the points $(0, 0)$ to $(2, 2)$; $(2, 2)$ to $(4, 2)$; and $(4, 2)$ to $(5, 0)$.

a) Sketch the graph of the velocity function.

b) Find a piecewise-defined formula for the location of the car at time t , for $0 \leq t \leq 5$ minutes.

c) Sketch its graph.

d) Use your formula for $x(t)$ from **b)** to determine how far the particle traveled between times $t = 0$ and $t = 5$.

e) Your answer to part **d)** should agree with a certain area associated to the graph of the velocity function. Explain this connection and why it is true.

f) What is the acceleration function $a(t)$ of the car in this problem?

Note: if you want graph paper, it's easy to find on-line, e.g. at <http://www.printfreegraphpaper.com>.

