Exam 2 Review Sheet
Math 2250-1 November 2012

Our exam covers chapters 4.1-4.4, 5, EP3.7, and 10.1-10.3 (also "resonance entries" from 10.4)of the text. Only scientific calculators will be allowed on the exam. The last page of the exam will be a xerox of the Laplace transform table from the front of the text.

Bring your ID's.

• remember to get to class 5 minutes early, and that you will be able to work five minutes late, for a total of one hour, from 8:30-9:30 am in JFB 103.

Chapter 4.1-4.4

At most 30% of the exam will deal directly with this material....but much of Chapter 5 uses these concepts, so much more than 30% of the exam will be related to chapter 4. (And, as far as matrix and determinant computations go, you should remember everything you learned in Chapter 3.)

Do you know the key definitions?

vector space

a collection of objects that you can add and scalar multiply, staying in your collection, so that the usual algebra properties (that you know for "regular" vectors) hold. Includes concept of **subspace** of a larger vector space.

linear combination of a collection $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$ of k vectors
any sum of scalar multiples of those vectors, i.e. any

$$\underline{\boldsymbol{v}} = c_1\underline{\boldsymbol{v}}_1 + c_2\underline{\boldsymbol{v}}_2 + \ldots c_k\,\underline{\boldsymbol{v}}_k$$

linearly independent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$
the only linear combination that adds up to the zero vector is the one for all the linear combo coefficients are zero.

$$c_1\underline{\boldsymbol{v}}_1 + c_2\underline{\boldsymbol{v}}_2 + \ldots c_k\,\underline{\boldsymbol{v}}_k = \underline{\boldsymbol{0}} \Rightarrow c_1 = c_2 = \ldots = 0$$

linearly dependent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$
not linearly independent.

span of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$
the collection of all possible linear combinations.

subspace of a vector space
a subset of that vector space that is itself a vector space, and this will be true precisely if the subset is closed under addition and scalar multiplication. (We called those conditions "alpha" and "beta".)

basis of a vector space
a collection of vectors that are linearly independent, and that span the vector space. In other words, every element of the vector space is a unique linear combination of those vectors.

how do you get a basis if you already have a (possibly dependent) spanning set?
delete dependent vectors until you get an independent spanning set.

how do you get a basis if you have an independent set that doesn't span the vector space?
add vectors not yet in the span of the ones you have, build up to a spanning (and still independent) set.

dimension of a vector space

the NUMBER of vectors in ANY basis for that vector space.

Subspace examples from Chapter 4, involving the concepts above
solutions to matrix equation $[A]\underline{x} = \underline{0}$ found general solution set by reducing the system and backsolving.....if we wrote the solution in linear combination form, then the vectors that appeared in the linear combination were always a basis.

span of vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_k$ found basis by deleting dependent vectors ... found the dependencies by putting those vectors into the columns of a matrix, reducing, and by seeing dependencies in the columns of the reduced matrix, which are actually also dependencies in the original matrix columns.

Subspace examples from Chapter 5
solutions to <u>homogeneous</u> <u>linear differential equation</u> for e.g. $y = y(x)$ on an interval I, i.e. solutions to

$$L(y) := y^{(n)} + p_{n-1}(x)\, y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = 0$$

What does it mean for a transformation $L : V \to W$ between vector spaces to be <u>linear</u>?
L is linear means

$$L(y_1 + y_2) = L(y_1) + L(y_2) \text{ for all vectors } y_1, y_2$$
$$L(c\,y) = c\,L(y) \text{ for all vectors } y \text{ and scalars } c.$$

Chapter 4 examples?

$$L(\underline{x}) = A\,\underline{x}$$

Chapter 5 examples?
The differential operator $L(y)$ above.

What is the general solution to $L(y) = f$, if L is a linear transformation (or "operator"), in terms of particular and homogeneous solutions?

$$y = y_P + y_H$$

where y_P is any particular solution and y_H is the general homogeneous solution.

Examples?

solution space to $[\boldsymbol{A}]\underline{\mathbf{x}} = \underline{\boldsymbol{b}}$

solution space to $L(y) = f$, i.e. the non-homogeneous linear DE.

Chapter 5 and EP3.7 (circuits).
At least 50% of the exam will be related to this material.

What is the **natural initial value problem** for n^{th}-order linear differential equation, i.e. the one that has unique solutions?

$$L(y) := y^{(n)} + p_{n-1}(x)y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = f(x)$$
$$y(x_0) = b_0$$
$$y'(x_0) = b_1$$
$$\vdots$$
$$y^{(n-1)}(x_0) = b_{n-1}$$

What is the **dimension of the solution space to the homogeneous DE**

$$L(y) := y^{(n)} + p_{n-1}(x)y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = 0 \ ?$$

dimension is "n"

How can you tell if solutions $y_1(x), y_2(x), \ldots y_n(x)$ are a **basis** for the homogeneous solution space above?

In the text they really like to compute the Wronskian determinant.

For IVP's it does make sense to make sure the Wronskian matrix has an inverse at x_0

We had other ways, using limits, plugging in x-values etc.

How is your answer above related to a **Wronskian matrix** and the **Wronskian determinant**?

How do you find the **general solution** to the **homogeneous constant coefficient linear DE**

$$L(y) := a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y' + a_0 y = 0 \ ?$$

(your answer should involve the characteristic polynomial, Euler's formula, repeated roots, complex roots.) Do you remember Euler's formula? Do you remember the Taylor-Maclaurin series formula in general? For e^x, $\cos(x)$, $\sin(x)$ in particular?

Try e^{rx} you get that r must be a root of characteristic polynomial $p(r)$.

$r = a + bi$ get solutions

$$e^{ax}\cos(bx), e^{ax}\sin(bx)$$

Euler:

$$e^{ib} = \cos(b) + i\sin(b)$$

What is another word for the "**principle of superposition**"?

Linearity.

What form does the **general solution to the non-homongeneous linear differential equation**

$$L(y) := y^{(n)} + p_{n-1}(x)y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = f(x)$$

take?

What three (?!) ways do you know to find **particular solutions to constant coefficient non-homogeneous linear DEs**

$$L(y) := a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y' + a_0 y = f?$$

method of undetermined coefficients.
variation of parameters.
Laplace transform.

5.4, 5.6, EP3.7 Mechanical vibrations and forced oscillations; electrical circuit analog

What are the governing second order DE's for a **damped mass-spring configuration** (Newton's second law) and for an **RLC circuit (**Kirchoff's Law for potential energy drop)?

What are **unforced undamped oscillations**, and their **solution formulas/behavior**?

Can you convert a linear combination $A\cos(\omega t) + B\sin(\omega t)$ in amplitude-phase form? Do you remember the addition angle formulas? Can you explain the physical properties of the solution?

$$m\,x''(t) + k\,x(t) = 0$$

$$A\cos(\omega_0 t) + B\sin(\omega_0 t);\ \omega_0 = \mathrm{sqrt}\left(\frac{k}{m}\right)$$

What are **unforced damped oscillations**, and their **solution formulas/behavior (three types)?**

$$m\,x''(t) + cx'(t) + k\,x(t) = 0 \quad c \text{ not zero}$$

What are the possible phenomena with **forced undamped oscillations** (assuming the forcing function is sinusoidal)?

beating, pure resonance, boring.

What are the possible phenomena with **forced damped oscillations** (assuming the forcing function is sinusoidal)?

boring (steady periodic plus transient), or practical resonance.

Can you solve all initial value problems that arise in the situations above?

Can you use **total energy in conservative systems, TE=KE+PE,** to derive the second order DE which is Newton's law, at least for examples we've discussed?

mass-spring, pendulum.

Chapter 10: Laplace transform techniques

At least 25% of the exam will deal directly with this material. The most efficient way for me to test it is to have you solve IVPs from chapter 5 (or earlier), using chapter 5 techniques as well as Laplace transform techniques. You will be responsible for 10.1-10.3 on the exam. (Sections 10.4-10.5 are yet to come.) You will be provided with the Laplace transform table on the front book cover and should be ready to use any table entry we've discussed. This includes the resonance entries, which the text does not cover until 10.4.

Can you use the **definition of Laplace transform** to compute Laplace transforms?

Can you **convert a constant coefficient linear differential equation IVP** into an algebraic expression for the **Laplace transform $X(s)$ of the solution $x(t)$**?

Can you use **partial fraction techniques** and **completing the square algebra** to break $X(s)$ into a linear combination of simpler functions for which the Laplace transform table will enable you to compute $\mathcal{L}^{-1}\{X(s)\}(t)$?