

Math 2250-1
Friday December 7
Course review

Final exam: Monday December 10 8:00-10:00 a.m. (I will let you work until 10:15). We will use our lecture room WEB L104 (here). As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. The algebra and math on the exam should all be doable by hand.

Review of previous final exam: Saturday December 8, 3-5 p.m. WEB L104 (here).

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters.

Chapters

- 1-2: 10-20% first order DEs
- 3-4: 20-30% matrix algebra and vector spaces
- 5, EP3.7: 15-30% linear differential equations and applications
- 6.1-6.2: 15-30% eigenvalues and eigenvectors, including complex case
- 7.1-7.4: 20-40% linear systems of differential equations and applications
- 9.1-9.4: 15-25% non-linear autonomous systems of DEs and applications
- 10.4-10.5, EP 7.6: 15-25% Laplace transform

On the next page is a more detailed list of the topics we've investigated this semester. They are more inter-related than you may have realized at the time, so let's discuss the connections. Then we'll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

slope fields
phase diagrams for autonomous DEs
equilibrium solutions
stability
existence-uniqueness thm for IVPs
methods:
separable
linear
applications
populations
velocity-acceleration models
input-output models

3-4 matrix algebra and vector spaces

linear systems and matrices
reduced row echelon form
matrix and vector algebra
manipulating and solving matrix-vector equations for unknown vectors or matrices.
matrix inverses
determinants
vector space concepts
vector spaces and subspaces
linear combinations
linear dependence/independence
span
basis and dimension
linear transformations
aka superposition
fundamental theorem for solution space to $L(y)=f$ when L is linear

5 Linear differential equations

IVP existence and uniqueness
Linear DEs
Homogeneous solution space, its dimension, and why
superposition, $\mathbf{x}(t) = \mathbf{x}_p + \mathbf{x}_h$
Constant coefficient linear DEs
 $\mathbf{x}_h$ via characteristic polynomial
Euler's formula, complex roots
 $\mathbf{x}_p$ via undetermined coefficients
solving IVPs
applications:
mechanical configurations
unforced: undamped and damped
cos and sin addition angle formulas and amplitude-phase form

forced undamped: beating, resonance
forced damped: $\mathbf{x}_{sp} + \mathbf{x}_{tr}$, practical resonance
RLC circuits
Using conservation of total energy ($=KE+PE$) to derive equations of motion, especially for mass-spring and pendulum

6.1-6.1 eigenvalues, eigenvectors (eigenspaces), diagonalizable matrices...including complex eigendata.

7.1-7.4 linear systems of DEs

first order systems of DEs and tangent vector fields.

existence-uniqueness thm for first order IVPs
superposition, $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$
dimension of solution space for $\mathbf{x}_h$.
conversion of DE IVPs or systems to first order system IVPs.

Constant coefficient systems and methods:

$$\mathbf{x}'(t) = A\mathbf{x}$$

$$\mathbf{x}'(t) = A\mathbf{x} + \mathbf{f}(t)$$

$$\mathbf{x}''(t) = A\mathbf{x} \quad (\text{from conservative systems})$$

$$\mathbf{x}''(t) = A\mathbf{x} + \mathbf{f}(t)$$

applications: phase portrait interpretation of unforced oscillation problems; input-output modeling; force and unforced mass-spring systems.

9.1-9.4 non-linear systems of differential equations

autonomous systems of first order DEs

equilibrium solutions

stability

phase portraits

linearization near equilibria, stability analysis, further classification and qualitative sketching.

Applications to interacting populations and non-linear mechanical configurations.

10.1-10.5, EP7.6: Laplace transform

definition, for direct computation

using table for Laplace and inverse Laplace transforms ... including for topics after the second midterm.

Solving linear DE (or system of DE) IVPs with Laplace transform.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

$$x''(t) + 5x'(t) + 4x(t) = 0$$

$$x''(t) + 5x'(t) + 4x(t) = 3\cos(2t)$$