Name________________________

Student I.D.___________________

Math 2250−1
Quiz 9
November 4, 2011

1) Consider the differential equation for $x(t)$, which could arise in a model for forced damped mechanical motion:

$$x''(t) + 2\,x'(t) + 5\,x(t) = 10\,\sin(t).$$

1a) Find a particular solution to this differential equation using the method of undetermined coefficients.

(6 points)

Try

$$\begin{aligned} 5\cdot[\ x(t) &= A\cdot\cos(t) + B\cdot\sin(t)] \\ +2\cdot[\ x'(t) &= -A\cdot\sin(t) + B\cdot\cos(t)\ ] \\ +1\cdot[\ x''(t) &= -A\cdot\cos(t) - B\cdot\sin(t)] \\ L(x) &= \cos(t)\cdot[5\,A + 2\,B - A] \\ &\quad + \sin(t)\cdot[5\,B - 2\,A - B] \end{aligned}$$

Since we want $L(x) = 10\,\sin(t)$ *we must have*

$$4\,A + 2\,B = 0$$
$$-2\,A + 4\,B = 10$$

i.e.

$$2\,A + B = 0$$
$$-A + 2\,B = 5$$

which has solution

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{5}\cdot\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

$$x_P(t) = -\cos(t) + 2\,\sin(t)$$

(no time on quiz, but could you put this into phase amplitude form?)

1b) What is the general solution to the differential equation above?

(3 points)

$x(t) = x_P(t) + x_H(t)$, need to find $x_H(t)$, the solutions to the homogeneous DE. The characteristic polynomial is $p(r) = r^2 + 2\,r + 5 = (r+1)^2 + 4$, so the roots are $r = -1 \pm 2\,i$ and

$$x_H(t) = c_1 e^{-t}\cdot\cos(2\,t) + c_2 e^{-t}\cdot\sin(2\,t) = e^{-t}\left(C_1\cdot\cos\left(2\,t - \alpha_1\right)\right).$$

1c) Why is the particular solution you found in part (1a) called the steady−periodic solution for this forced and damped oscillation problem?

(1 point)

Because as $t \to \infty$*, the transient homogeneous solution* $x_H(t) = x_{tr}(t)$ *converges to zero, so regardless of initial conditions,*

$$x(t) = x_P(t) + x_H(t) \to x_P(t),\ \text{as}\ t \to \infty\ .$$