Name__________________________

Student I.D.____________________

Math 2250−1
Quiz 8
October 28, 2011
SOLUTIONS

1) Consider the differential equation for $x(t)$, which could arise in a model for damped mechanical motion:

$$x''(t) + 2\cdot x'(t) + 5\cdot x(t) = 0 .$$

1a) Find the general solution to this differential equation.

(6 points)

$p(r) = r^2 + 2\cdot r + 5$. We wish to solve $p(r) = 0$:

$$r^2 + 2\cdot r + 5 = 0$$
$$(r+1)^2 + 4 = 0$$
$$(r+1)^2 = -4$$
$$r + 1 = \pm 2\cdot i$$
$$r = -1 \pm 2\cdot i .$$

Of course, we could also use the quadratic formula, $r = -\frac{2 \pm \sqrt{4-20}}{2}$, which simplifies to the same answer. Thus the general solution for $x = x(t)$ is

$$x(t) = c_1\cdot e^{-t}\cdot\cos(2\cdot t) + c_2\cdot e^{-t}\cdot\sin(2\cdot t) .$$

1b) What kind of damping is illustrated in this differential equation and its solutions?

(1 point)

underdamped, because the solution oscillates through x=0 infinitely often....this happens whenever the roots of the characteristic polynomial are complex.

1c) Use your work in (1a) to solve the initial value problem

$$x'' + 2\cdot x' + 5\cdot x = 0$$
$$x(0) = -1$$
$$x'(0) = 3 .$$

(3 points)

$$x(t) = c_1\cdot e^{-t}\cdot\cos(2\cdot t) + c_2\cdot e^{-t}\cdot\sin(2\cdot t)$$
$$x'(t) = c_1\cdot\left[-e^{-t}\cdot\cos(2\cdot t) + e^{t}\cdot(-2)\cdot\sin(2\cdot t)\right] + c_2\cdot\left[-e^{-t}\cdot\sin(2\cdot t) + 2\cdot e^{-t}\cdot\cos(2\cdot t)\right].$$
$$x(0) = -1 = c_1$$
$$x'(0) = 3 = -c_1 + 2\cdot c_2 .$$

The first equation implies $c_1 = -1$, so from the second equation we deduce $c_2 = 1$.

$$x(t) = e^{-t}\cdot(-\cos(2\cdot t) + \sin(2\cdot t))$$

If you'd had more time for the quiz, I could have asked for this solution to be converted into time−varying amplitude−phase form,

$$x(t) = C\cdot e^{-t}\cdot\left(\cos\left(2\,t - \alpha\right)\right) .$$