Name_________________________

Student I.D.____________________

Math 2250−1
Quiz 7
October 21, 2011
SOLUTIONS

1) Consider the differential equation for $y(x)$

$$y'' + 5\cdot y' + 6\cdot y = 0 .$$

1a) Find the general solution to this differential equation.

(6 points)

trying $y(x) = e^{r\cdot x}$ *yields* $L(y) = e^{r\cdot x}\cdot\left(r^2 + 5\cdot r + 6\right)$ *so in order for* $e^{r\cdot x}$ *to be a solution, r must be a root of the characteristic polynomial*

$$r^2 + 5\cdot r + 6 = (r+3)\cdot(r+2) .$$

Since the roots are r = -3, -2 the general solution is

$$y_H(x) = c_1\cdot e^{-3\cdot x} + c_2\cdot e^{-2\cdot x} .$$

1b) What is the dimension of the solution space above?

(1 point)

Since there are two basis functions, $y_1 = e^{-3\cdot x}, y_2 = e^{-2\cdot x}$ *for the solution space, the dimension is two.*

1c) Use your work in (1a) to solve the initial value problem

$$y'' + 5\cdot y' + 6\cdot y = 0$$
$$y(0) = -1$$
$$y'(0) = 4 .$$

(3 points)

$$y(x) = c_1\cdot e^{-3\cdot x} + c_2\cdot e^{-2\cdot x}$$
$$y'(x) = -3\cdot c_1\cdot e^{-3\cdot x} - 2\cdot c_2\cdot e^{-2\cdot x}$$

at x = 0 :

$$-1 = c_1 + c_2$$
$$4 = -3\cdot c_1 - 2\cdot c_2 .$$

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix}^{-1}\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \frac{1}{1}\cdot\begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} .$$

So

$$y(x) = -2\cdot e^{-3\cdot x} + e^{-2\cdot x} .$$