Name________________________

Student I.D.__________________

Math 2250-1
Quiz 6 SOLUTIONS
October 7, 2011

1) What two properties must a subset W of a vector space V satisfy in order to be a subspace of V?

(2 points)

W must be closed under addition and scalar multiplication

2) Consider the unit circle in R^2 , i.e. $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in R^2 \text{ such that } x^2 + y^2 = 1 \right\}$. Give a reason to explain why W is not a subspace of R^2.

(2 points)

Any of the following explanations would do:
(i) W does not contain the zero vector (as it must, since if W is closed under scalar multiplication than it contains $\underline{\mathbf{0}} = 0 \cdot \underline{\boldsymbol{w}}$ *for any* $\underline{\boldsymbol{w}} \in W$.
(ii) W is not closed under addition: for example $[0, 1]$ *and* $[1, 0]$ *are in W since they satisfy the condition* $x^2 + y^2 = 1$ *, but* $[1, 0] + [0, 1] = [1, 1]$ *is not in W since* $1^2 + 1^2 = 2$.
(iii) W is not closed under scalar multiplication, since $2 \cdot [0 , 1] = [0, 2]$ *is not in W, even though* $[0, 1]$ *is.*

3a) The solution set in R^3 of the equation

$$x + y + 3 \cdot z = 0$$

is a subspace of R^3. Find a basis for this subspace. (You don't need to justify your answer on this short time limit quiz, but do show your work.)

(5 points)

The augmented matrix for this very small system is $[1\ 1\ 1 \mid 0]$, *which is already in reduced row-echelon form. If we backsolve we get* $z = t,\ y = q,\ x = -q - t$ *. In vector form this is*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = q \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Thus

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is a basis (by construction these two vectors span, and it's also easy to check that they are linearly independent).

A quicker way to do this problem is to just find two non-zero vectors in this plane which are not multiples *of each-other, since we know the plane is two dimensional, and for just two vectors linear independence is the same as not being scalar multiples....this would quickly lead you to two vectors like the ones above, if you set one of the coordinates equal to zero.*

3b) What is the dimension of the subspace in (3a).

(1 point)

the dimension is 2, because it takes 2 vectors to make a basis.