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Student I.D._____________________

Math 2250−1
Quiz 5 Solutions
September 23, 2011

1) Consider the following system of equations

$$2\cdot x + y + 3\cdot z = 4$$
$$x + y - z = 3$$
$$-x - 2\cdot y + 6\cdot z = -5$$

1a) Exhibit the augmented matrix corresponding to this system, compute its reduced row echelon form, and find all solutions to the system.

(7 points)

$$A := \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 1 & -1 & 3 \\ -1 & -2 & 6 & -5 \end{bmatrix}$$

$R2 \rightarrow R1,\ R1 \rightarrow R2$:

$$\begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & 1 & 3 & 4 \\ -1 & -2 & 6 & -5 \end{bmatrix}$$

$R2 \rightarrow -2\cdot R1 + R2,\ R3 \rightarrow R3 + R1$:

$$\begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & -1 & 5 & -2 \\ 0 & -1 & 5 & -2 \end{bmatrix}$$

$R2 \rightarrow -R2,\ R3 \rightarrow -R2 + R3$:

$$\begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R1 \rightarrow -R2 + R1$:

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus z can be a free parameter, $z = t$, $y = 2 + 5\cdot t$, $x = 1 - 4\cdot t$, *or in vector linear combination form*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t\cdot \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}.$$

This is the parametric equation for a line through $[1, 2, 0]$ *and with velocity vector* $[-4, 5, 1]$).

1b) If we interpret solutions to this system as the intersection set of three planes, what geometric configuration of the planes does this system exhibit?

(1 point)

From the solution above we see that the solution set to this system represents three planes intersecting in a line.

2) Here are two matrices A and B . Only one of the matrix products AB, BA exists. Compute this product.

$$A := \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix} \qquad B := \begin{bmatrix} -2 & 2 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Only BA makes sense, and

$$BA = \begin{bmatrix} -2 & 2 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ -1 & 4 \\ 0 & 17 \end{bmatrix}$$

(2 points)