

Name \_\_\_\_\_  
Student I.D. \_\_\_\_\_

**Math 2250-1**  
**Quiz 4 Solutions**  
**September 16, 2011**

1) Consider the following linear drag initial value problem:

$$\frac{dv}{dt} = 4 - 0.1 \cdot v .$$
$$v(0) = 0 .$$

1a) Use a phase diagram to determine the limit of the solution  $v(t)$  to this problem, as  $t \rightarrow \infty$ .

(2 points)

The (constant) equilibrium solution satisfies  $4 - 0.1 \cdot v = 0$ , i.e.  $v = 40$ . When  $v > 40$ ,  $\frac{dv}{dt} < 0$ , and when  $v < 40$ ,  $\frac{dv}{dt} > 0$ . Thus the phase diagram is:

$$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow 40 \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow$$

so all solutions converge to the asymptotically stable solution  $v = 40$  as  $t \rightarrow \infty$ .

1b) Solve the initial value problem above. (Your solution should be consistent with the correct answer to part (a) above.)

(6 points)

This DE is separable and linear. You may solve it either way, of course, but I find linear easier for these problems:

$$\frac{dv}{dt} + 0.1 \cdot v = 4$$
$$e^{0.1 \cdot t} \cdot \left( \frac{dv}{dt} + 0.1 \cdot v \right) = 4 \cdot e^{0.1 \cdot t}$$
$$\frac{d}{dt} (e^{1 \cdot t} \cdot v) = 4 \cdot e^{0.1 \cdot t}$$
$$e^{1 \cdot t} \cdot v = 40 \cdot e^{1 \cdot t} + C$$
$$v(0) = 0 \Rightarrow C = -40, \text{ so}$$
$$e^{1 \cdot t} \cdot v = 40 \cdot e^{1 \cdot t} - 40$$
$$\text{divide by } e^{1 \cdot t} \Rightarrow$$
$$v = 40 - 40 \cdot e^{-1 \cdot t} .$$

1c) Use Newton's Law to convert the following information into a differential equation initial value problem – your IVP should end up being equivalent to the IVP at the top of this page, if you keep track of your units correctly.

A small motor boat and its passenger/driver weigh a total of 640 lbs. The motor provides a thrust of 80 lbs. The drag from the boat depends on the velocity  $v$  of the boat, and is 2 lbs for each ft/sec of velocity  $v$ . Aside from the motor and the drag forces, there are no other forces acting on the boat, and the boat

starts from rest.

(2 points)

weight is a force i.e.  $W = mg$ . Since  $g = 32 \frac{\text{ft}}{\text{s}^2}$  in the English unit system, we deduce that the mass of the

boat is given by  $m = \frac{W}{g} = \frac{640}{32} = 20$  slugs. So Newton's law is

$$20 \cdot \left( \frac{dv}{dt} \right) = 80 - 2 \cdot v.$$

Dividing both sides by 20 yields the DE at the top of this page. Since the boat starts at rest,  $v(0) = 0$ .