Name________________________
Student I.D.___________________

Math 2250−1
Quiz 2 Solutions
September 2, 2011

1) Solve the following initial value problem.

$$x'(t) + 0.02\cdot x(t) = 1$$
$$x(0) = 0$$

(7 points)

This differential equation is both linear and separable. You can solve it either way. Probably most people solved it as a linear DE:
The integrating factor is

$$e^{\int P(t)\cdot dt} = e^{0.02\cdot t} \ ,$$

so:

$$e^{0.02\cdot t}\cdot(x'(t) + 0.02\cdot x(t)) = e^{0.02\cdot t}$$
$$\frac{d}{dt}\left(e^{0.02\cdot t}\cdot x(t)\right) = e^{0.02\cdot t}$$
$$e^{0.02\cdot t}\cdot x(t) = \int e^{0.02\cdot t}\cdot dt = 50\cdot e^{0.02\cdot t} + C$$
$$\textit{divide by } e^{0.02\cdot t} \Rightarrow x(t) = 50 + C\cdot e^{-0.02\cdot t}$$
$$x(0) = 0 \Rightarrow 0 = 50 + C \Rightarrow C = -50.$$

Thus

$$\mathbf{x(t) = 50 - 50\cdot e^{-0.02\cdot t}} \ .$$

2) A tank contains 100 l of water, which is initially pure. At time $t = 0$ valves are opened, so that a salt−water solution flows into the tank at a rate of $2\ \frac{l}{s}$, with a salt concentration of $0.5\ \frac{kg}{l}$; at the same time, well−mixed water begins to flow out of the tank at the same rate of $2\ \frac{l}{s}$, maintaining the constant volume 100 l of water in the tank. Let $x(t)\ kg$ be the mass of (dissolved) salt in the tank at time t. Use input−output modeling to show that $x(t)$ satisfies the initial value problem in (1).

(3 points)

$$\mathbf{\frac{dx}{dt} = r_i\cdot c_i - r_o\cdot c_o}$$

We see that $r_i = r_o = 2$, $c_i = 0.5$ and $c_o = \frac{x}{V} = \frac{x}{100}$ so

$$\frac{dx}{dt} = 2\cdot(0.5) - \frac{2\cdot x}{100} = 1 - 0.02\cdot x.$$
$$\mathbf{x'(t) + 0.02\cdot x(t) = 1} \ .$$

Since there is initially no salt in the tank water, $\boldsymbol{x(0) = 0}$.