Name_____
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Math 2250−1
Quiz 1 Solutions
August 26, 2011

1) Write down an initial value problem for the function N(t), as described below. Do not attempt to find the actual solution function.

In a city with a population of 10 thousand people the **time rate of change of the number** N of those persons infected with a certain contagious disease **is proportional to** the **product** of the **number who have the disease** and **the number who do not**. At time $t = 0$ days there are 2 thousand infected people. (4 points)

translating the underlined words into a differential equation yields

$$\frac{dN}{dt} = k \cdot N \cdot (P - N) \ ,$$

where P = 10, 000. The initial condition is that $P(0) = 2000$. *Depending on your choice of units for population you could have two equivalent IVPs:*

If your population units are individual people then the IVP is

$$\frac{dN}{dt} = k \cdot N \cdot (10,000 - N)$$
$$P(0) = 2000.$$

If you choose to count people by the thousands, then the IVP is

$$\frac{dN}{dt} = k \cdot N \cdot (10 - N)$$
$$P(0) = 2.$$

(Both versions of the IVP are correct. Sometimes picking appropriate units makes solving differential equations computationally simpler.)

2) Find the position function $x(t)$ of a moving particle with the acceleration $a(t) = 8 \cdot \sin(2 \cdot t) \ \frac{meters}{\sec^2}$, given that its initial position is $x(0) = 8$ *meters* and its initial velocity is $v(0) = -4 \ \frac{meters}{\sec}$. (6 points)

$$x''(t) = 8 \cdot \sin(2 \cdot t) \ ;$$
$$x'(t) = \int 8 \cdot \sin(2 \cdot t) dt = -4 \cdot \cos(2 \cdot t) + C \ ;$$
$$v_0 = -4 \cdot \cos(0) = -4 + C \ , \text{ so } C = 0.$$
$$x'(t) = -4 \cdot \cos(2 \cdot t) \ ,$$
$$x(t) = \int -4 \cdot \cos(2 \cdot t) \ dt = -2 \cdot \sin(2 \cdot t) + C \ ,$$
$$x_0 = 8 = -2 \cdot \sin(0) + C = C \ , \text{ so } C = 8 \ , \text{ so}$$
$$x(t) = -2 \cdot \sin(2 \cdot t) + 8 \quad meters$$