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Student I.D.____________________

Math 2250−1
Quiz 13 SOLUTIONS
December 9, 2011

1) Consider the first order system of differential equations

$$x'(t) = y$$
$$y'(t) = -4\sin(x) - 5y.$$

1a) Find the equilbrium solutions.

(2 points)

Equilibrium solutions are constant solutions so we must simultaneously have $y = 0,\ -4\sin(x) - 5y = 0$ *, which is equivalent to* $y = 0,\ \sin(x) = 0$ *.*
Thus the equilibrium solutions are $(n\pi, 0)$ *where n is ranges through the integers. (You might recognize this first order system as possibly arising from a damped pendulum problem.)*

1b) Linearize the first order system above at the equilibrium solution $(x, y) = (0, 0)$. Classify this critical point, making sure to include whether or not it is stable. (If there was more time and it was asked, could you draw a qualitatively accurate phase portrait near this point?)

(8 points)

In general if (x_e, y_e) *is an equilibrium solution and if we write* $x(t) = x_e + u(t), y(t) = y_e + v(t)$ *then the linearized equation for* $[u(t), v(t)]^T$ *uses the Jacobian matrix for* $[F(x, y), G(x, y)]^T$ *:*

$$J(x, y) = \begin{bmatrix} \frac{\partial}{\partial x}F & \frac{\partial}{\partial y}F \\ \frac{\partial}{\partial x}G & \frac{\partial}{\partial y}G \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4\cos(x) & -5 \end{bmatrix}.$$

$$J(0, 0) = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}.$$

So the linearized system is

$$\begin{bmatrix} u'(t) \\ v'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\begin{bmatrix} u(t) \\ v(t) \end{bmatrix}$$

$$|J - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -4 & -5-\lambda \end{vmatrix} = \lambda(\lambda + 5) + 4 = \lambda^2 + 5\lambda + 4 = (\lambda + 4)(\lambda + 1) = 0$$

So the eigenvalues are $\lambda = -4, -1$ *. Thus* $(0, 0)$ *is a stable (improper) node, aka nodal sink. Could you could find the eigenbasis for the Jacobian matrix, the general solution to the linearized problem, and could you sketch a qualitatively accurate phase portrait for the linearized system?*