

Name _____

Student I.D. _____

Math 2250-1
Quiz 11
November 23, 2011

1a) Find the general solution $[x(t), y(t)]^T$ to the homogeneous system of differential equations

$$\begin{aligned}x'(t) &= -4x + 2y \\y'(t) &= 4x - 2y.\end{aligned}$$

(6 points)

1b) Consider two tanks. Tank one contains 50 gallons of water, and tank two contains 100 gallons of water. Water flows from tank 1 to tank 2 through a one pipe, and from tank 2 back to tank 1 through another pipe. The flow rate in both pipes is 200 gallons per hour. There are no other inlet or outlet pipes. After some initial allocation of solute $x(0) = x_0$ to tank 1 and $y(0) = y_0$ to tank 2, let $x(t)$ and $y(t)$ denote the salt amounts for $t > 0$. Assume the water in each tank is well-mixed so that salt concentrations can be treated as uniform in each tank. Use this information and input-output analysis to derive the first order system of differential equations for $x(t)$ and $y(t)$. (Your answer is the system in part (a).)

(4 points)