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Math 2250−1
Quiz 10 solutions
November 18, 2011

1) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}.$$

1a) Find the eigenvalues of A, and corresponding eigenvectors.

(8 points)

$$\left|A - \lambda \cdot I\right| = \begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = (\lambda - 1)(\lambda - 3) - 8 = \lambda^2 - 4\lambda - 5 = (\lambda - 5)\cdot(\lambda + 1)$$

so the eigenvalues are $5, -1$.

$\lambda = -1$:

$$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 4 & 4 & 0 \end{array}\right]$$

Since $-1 \cdot col_1 + 1 \cdot col_2 = 0$, *we deduce that* $\underline{\mathbf{v}} = [-1, 1]^T$ *is an eigenvector.*

$\lambda = 5$:

$$\left[\begin{array}{cc|c} -4 & 2 & 0 \\ 4 & -2 & 0 \end{array}\right]$$

Since $col_1 + 2 \cdot col_2 = 0$ *we deduce that* $\underline{\mathbf{v}} = [1, 2]^T$ *is an eigenvector.*

1b) Is the matrix A diagonalizable? Explain.

(2 points)

Yes. Any of the following explanations suffice:
i) The two eigenvectors above are a basis for R^2 .
ii) For the two eigenvectors above, put into the columns of the invertible matrix P *we have* $AP = PD$ *where* D *is the diagonal matrix of corresponding eigenvalues:*

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$$

iii) Any n *by* n *matrix with* n *distinct eigenvalues is automatically diagonalizable, because the* n *corresponding eigenvectors are automatically linear independent and hence a basis for* R^n .