Math 2250−1
Week 9 concepts and homework, due October 28.

Recall that problems which are underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

5.3: using the algorithm for finding the general solution to constant coefficient linear homogeneous differential equations: real roots, Euler's formula and complex roots, repeated roots; solving associated initial value problems.

5.3: 3, **10, 14,** 21, **22, 24**, 27, **38**

w9.1) Euler's formula
$$e^{i\theta} = \cos(\theta) + i \cdot \sin(\theta)$$
is extremely useful in higher mathematics/science/engineering. Luckily, the rule of exponents works for such expressions. In other words
$$e^{i\alpha + i\beta} = e^{i\alpha} \cdot e^{i\beta} .$$
Rewrite the left side of this identity as $e^{i \cdot (\alpha + \beta)}$ and then use Euler's formula to expand both sides of the claimed equality above, in order to verify it. You'll notice that the identity is true because of the addition angle formulas for $\cos(\alpha + \beta)$ and $\sin(\alpha + \beta)$ (and so gives a good way to recover these identities if you happen to forget them).

5.4: mechanical vibrations: setting up and solving differential equations and initial value problems for mechanical vibrations; underdamped, critically damped, and overdamped vibration problems; applications to pendulums, mass−spring configurations, and related problems; understanding and using phase−amplitude form for sinusoidal functions.

5.4: 3, **4**, 5, **6, 8, 10**, 15, 17, 21, **23**. Note: in **10** the weight of the buoy is $m \cdot g = \rho \cdot V \cdot g = \rho \cdot \pi \cdot r^2 \cdot h \cdot g$ The text forgot to include the density ρ in computing the mass from the volume.
w9.2) Consider an unforced vibrating mass−spring system in which the mass is 2 *kg* , the spring has Hooke's constant $50 \frac{N}{m}$ and we are able to vary the damping coefficient c . Thus the differential equation governing the displacement $x(t)$ *m* from equilibrium is
$$2 \cdot x''(t) + c \cdot x'(t) + 50 \cdot x(t) = 0.$$
As we vary c we will keep the initial values the same:
$$x(0) = 2$$
$$x'(0) = 5 .$$
w9.2a) Solve the initial value problem when there is no damping, i.e. $c = 0$. Convert you solution to phase−amplitude form. Use Maple or other software to create a graph which shows about two periods of your solution, starting at $t = 0$, and print out a copy. Draw on this graph to identify the amplitude, period, and phase delay for this sinusoidal function.
w9.2b) Now consider an underdamped configuration, when $c = 4$. Solve the same initial value problem as before. Convert your solution into pseudo−amplitude, pseudo−phase form. Create and print out a graph of the solution which includes about two pseudoperiods, and indicate one of these on your graph. It's fine to use decimal approximations in this problem.
w9.2c) What value of c leads to critical damping? Solve the IVP for this value of c, and graph the solution on an interval of appropriate length.
w9.2d) Finally, consider the overdamped situation, when $c = 52$. Solve the initial value problem in this case, and create a graph of the solution.