Recall that problems which are underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

9.1 concepts for autonomous first order systems of DE's: equilibrium solutions, stability, introduction to the names for different types of equilibrium solutions, visualization.
9.1: 5, 8, 11, 15.

9.2 finding equilibrium solutions (aka critical points), and linearizing at the critical point in order to classify it and understand how the non–linear system behaves nearby.
9.2: 5, 7, 9, 13, 15, 17, 19
w15.1) Consider the non–linear system of differential equations from the competition model we discussed in class on Nov 30 and Dec 2:

$$x'(t) = 14\,x - 2\,x^2 - x\,y$$
$$y'(t) = 16\,y - 2\,y^2 - x\,y\,.$$

We found the four equilibrium (critical point) solutions $(0, 0)$, $(7, 0)$, $(0, 8)$, $(4, 6)$ and did a linearization analysis at $(4, 6)$, which is a stable node.
a) Linearize the system at the $(0, 8)$ using the Jacobian matrix, and classify the critical point according to the eigenvalues of the matrix in the linearized system, using table 9.2.12 on page 539.
b) Compute the general solution to the linearized system at $(0, 8)$, and use the eigenvalue/eigenvector information to sketch a qualitatively accurate phase portrait in the $u - v$ plane, like we did in class on Friday and Monday for the equilibrium points $(4, 6)$ and $(7, 0)$. (Your picture should look very much like the pplane solution to the non–linear problem near $(u, v) = (0, 0)$, i.e. near $(x, y) = (0, 8)$.

9.3: interacting population models: competition and predator–prey modeling
9.3: 5, 7, 11, **13**,
w15.2) We and the text discuss the fact that for the general logisitic competition model

$$x'(t) = a_1 \cdot x - b_1 \cdot x^2 - c_1 \cdot x\,y$$
$$y'(t) = a_2 \cdot y - b_2 \cdot y^2 - c_2 \cdot x\,y\,,$$

if the product $b_1 b_2$ satisifies $b_1 b_2 > c_1 c_2$ then any first quadrant equilibrium will be asymptotically stable and a global attractor, as in the system for **w15.1** . On the other hand, if $b_1 b_2 < c_1 c_2$ then one of the populations will "always" die out. This is illustrated in the following system:

$$x'(t) = 9\,x - x^2 - 2\,x\,y$$
$$y'(t) = 12\,y - y^2 - 2\,x\,y\,.$$

a) Find all equilibrium solutions.
b) Linearize the system about each of the 4 equilibrium solutions, and classify each equilibrium point according to the matrix eigenvalues.
c) For the equilibrium solution $(0, 12)$, find the general solution to the linearized problem, and draw a qualitatively correct (u, v) plane picture using this general solution (your picture will be like the one we drew on Friday Dec 2).
d) Use pplane to check your work in this problem: Create a phase portrait with the equilibrium solutions highlighted along representative solution curves. Include the stable and unstable orbits (also known as "seperatrices") from the saddle point in the first quadrant. Print out and hand in a screenshot of this diagram. (You can check your linearization analyses in a,b,c as well, but you don't need to hand this part in.) Explain how the location of the initial population point (x_0, y_0) determines which population

dies out and which one survives.

9.4) Nonlinear mechanical systems.
9.4: 9, **10**, 11, 13, **14**
w15.3) Consider the first order system on top of page 7 of Dec 2 notes,
$$x'(t) = y$$
$$y'(t) = -\sin(x) - 0.2\,y\,.$$
Notice this arises from the rigid rod pendulum equation $\theta''(t) + .2\,\theta'(t) + \sin(\theta(t)) = 0$, with
$x(t) = \theta(t)$, so we have taken the numerical value of $g = L = 9.8$ *m*, and added some friction. We've
discussed (or will discuss) the no–damping case in class.
a) What are the equilibrium solutions of the first order system? What swing positions do they
correspond to?
b) Linearize this system at $(\pi, 0)$. Compute the general solution to the linearized problem, classify the
critical point, and sketch a qualititatively accurate picture of the phase portrait for the linearized
problem. Use pplane to recreate the phase portrait for this system on a suitable scale, create
representative solution trajectories, and plot the stable and unstable orbits from $(\pi, 0)$. At which two
possible equilibrium points will solutions that start near to $(\pi, 0)$ end up, with $100\,\%$ probability, based
on the phase portrait for the non–linear problem? What is the interpretation of this in terms of the rigid–
rod swing?
c) Linearize the system at $(0, 0)$, classify the critical point, and sketch a qualititatively correct picture of
the linearized problem near there. (As we discuss in class, you need not find the actual eigenvectors –
you can just see what the tangent field looks like near $(0, 0)$ to deduce the direction the spiral is rotating.
)
d) Indicate on your pplane plot the region of initial values in the phase plane for which solutions
ultimately converge to $(0, 0)$.

w15.4) This is a continuation of **14** . Use the $\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)}$ trick and separation of variables to get the
(multiple of) total energy function for which the solution trajectories are following the level curves. Use
software to graph the total energy function, and verify visually that the level curves do correspond to the
solution trajectories as shown in figure 9.4.12. Hand in a picture of this graph.